1. Use Green's Theorem and/or a computer algebra system to evaluate $\int_C P\ dx + Q\ dy$,

where $P(x, y) = x^4 y^5,\ \ Q(x, y) = -x^7 y^6$, and C is the circle $x^2 + y^2 = 4$.

Select the correct answer.

a. $624\pi^2$ b. $65{,}640\pi$ c. $-32{,}977\pi$ d. -104π e. 208

2. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C (4xy)\ dx + (4x^2)\ dy$$

C consists of the line segment from (-3, 0) to (3, 0) and the top half of the circle $y^2 + x^2 = 9$.

Select the correct answer.

a. 0 b. 8 c. 16 d. 5.333333 e. 18

3. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C \mathbf{F}\ d\mathbf{r}\ ,\ \mathbf{F}(x, y) = (y^2 - x^2 y)\mathbf{i} + xy^2\mathbf{j}$$

C consists of the circle $x^2 + y^2 = 16$ from (4, 0) to (2, 2) and the line segments from (2, 2) to (0, 0) and from (0, 0) to (4, 0).

Select the correct answer.

a. $I \approx 37.77$ b. $I \approx 42.71$ c. $I \approx 41.39$ d. $I \approx 35.43$ e. $I \approx 32.43$

4. Use Green's Theorem to find the work done by the force $\mathbf{F}(x, y) = x(x + 5y)\mathbf{i} + 4xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to (4, 0) then along the line segment to (0, 4) and then back to the origin along the y-axis.

Select the correct answer.

a. -6 b. 32 c. -32 d. 192 e. -192

5. A particle starts at the point (-3, 0), moves along the x-axis to (3, 0) and then along the semicircle $y = \sqrt{9 - x^2}$ to the starting point. Use Green's Theorem to find the work done on this particle by the force field $\mathbf{F}(x, y) = \left\langle 24x,\ 8x^3 + 24xy^2 \right\rangle$.

Select the correct answer.

a. 486π b. 972π c. 486 d. 0 e. 48π

6. A plane lamina with constant density $\rho(x, y) = 12$ occupies a region in the xy-plane bounded by a simple closed path C. Its moments of inertia about the axes are $I_x = -\frac{\rho}{3}\int_C y^3\ dx$ and $I_y = \frac{\rho}{3}\int_C x^3\ dy$.

Find the moments of inertia about the axes, if C is a rectangle with vertices (0, 0), (4, 0), (4, 5) and (0, 5).

Select the correct answer.

a. (2,000, 1,280) b. (-1,280, 1,280) c. (2,000, -1,280) d. (-2,000, 1,280) e. (-2,880, 1,200)

7. Find the curl of the vector field.

$\mathbf{F}(x, y, z) = 2xy\mathbf{i} + 10yz\mathbf{j} + 5xz\mathbf{k}$

Select the correct answer.

a. $-10y\mathbf{i} - 5z\mathbf{j} - 2x\mathbf{k}$
b. $-5y\mathbf{i} - 10z\mathbf{j} - 2x\mathbf{k}$
c. $-2y\mathbf{i} - 5z\mathbf{j} - 10x\mathbf{k}$
d. $2y\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$
e. none of these

8. Find the correct identity, if f is a scalar field, **F** and **G** are vector fields.

Select the correct answer.

a. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{div}\,\mathbf{F} + \text{div}\,\mathbf{G}$

b. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{curl}\,\mathbf{F} + \text{div}\,\mathbf{G}$

c. $\text{div}(\mathbf{F} + \mathbf{G}) = \text{curl}\,\mathbf{F} + \text{curl}\,\mathbf{G}$

d. $\text{curl}(\mathbf{F} + \mathbf{G}) = \text{div}\,\mathbf{F} + \text{curl}\,\mathbf{G}$

e. none of these

9. Find the correct identity, if f is a scalar field, **F** and **G** are vector fields.

Select the correct answer.

a. $\text{div}(f\,\mathbf{F}) = f\,\text{curl}(\mathbf{F}) + (\nabla f) \times \mathbf{F}$

b. $\text{div}(f\,\mathbf{F}) = f\,\text{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$

c. $\text{curl}(f\,\mathbf{F}) = f\,\text{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$

d. none of these

10. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^{-7x}\mathbf{i} + e^{4y}\mathbf{j} + e^{5z}\mathbf{k}$ and C is the boundary of the part of the plane $8x + y + 8z = 8$ in the first octant.

Select the correct answer.

a. 0 b. 23 c. 49 d. 16 e. 69

11. Find a parametric representation for the part of the elliptic paraboloid $x + y^2 + 6z^2 = 9$ that lies in front of the plane $x = 0$.

Select the correct answer.

a. $x = x,\ \ y = \pm\sqrt{9 - x + 6z^2},\ \ z = z$
b. $x = x,\ \ y = \sqrt{9 - x + 6z^2},\ \ z = z$
c. $x = 9 - y^2 - 6z^2,\ \ y = y,\ \ z = y,\ \ 0 \le y^2 + 6z^2 \le 3$
d. $x = 9 - y^2 - 6z^2,\ \ y = y,\ \ z = y,\ \ y^2 + 6z^2 \ge 9$
e. $x = 9 - y^2 - 6z^2,\ \ y = y,\ \ z = y,\ \ y^2 + 6z^2 \le 9$

12. Let S be the cube with vertices $(\pm 1,\ \pm 1,\ \pm 1)$. Approximate $\iint_S \sqrt{x^2 + 2y^2 + 7z^2}$ by using a Riemann sum as in Definition 1, taking the patches S_{ij} to be the squares that are the faces of the cube and the points P_{ij} to be the centers of the squares.

Select the correct answer.

a. $4(1 + \sqrt{2} + \sqrt{7})$ b. $8(1 + \sqrt{2} + \sqrt{7})$ c. $8(3 + \sqrt{7})$ d. $8(3 + \sqrt{2})$ e. none of these

13. Suppose that $f(x, y, z) = g\left(\sqrt{x^2 + y^2 + z^2}\right)$ where g is a function of one variable such that $g(2) = 3$. Evaluate $\iint_S f(x, y, z)\ dS$ where S is the sphere $x^2 + y^2 + z^2 = 4$.

Select the correct answer.

a. 24π b. 48π c. 12π d. 18π e. none of these

14. Evaluate the surface integral where S is the surface with parametric equations $x = 7uv$, $y = 6(u + v)$, $z = 6(u - v)$, $u^2 + v^2 = 3$.

$$\iint_S 10yz\ dS$$

Select the correct answer.

a. $I = 10\pi$ b. $I = 3$ c. $I = 0$ d. $I = 10 + 7\pi$ e. $I = 10 + 3\pi$

15. The temperature at the point (*x, y, z*) in a substance with conductivity $K = 9$ is $u(x, y, z) = 6x^2 + 6y^2$.

Find the rate of heat flow inward across the cylindrical surface $y^2 + z^2 = 3,\ 0 \le x \le 10$.

Select the correct answer.

a. 6,480 b. $6{,}480\pi$ c. 648π d. $1{,}620\pi$ e. 324π

16. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

$\mathbf{F}(x, y, z) = 7xy\mathbf{i} + 7e^z\mathbf{j} + 7xy^2\mathbf{k}$

S consists of the four sides of the pyramid with vertices (0, 0, 0), (3, 0, 0), (0, 0, 3), (3, 0, 3) and (0, 3, 0) that lie to the right of the *xz*-plane, oriented in the direction of the positive *y* - axis.

Select the correct answer.

a. 12 b. 16 c. 49 d. 0 e. 1

17. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of **F** across *S*.

$\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + y^3\mathbf{k}$, *S* is the sphere $x^2 + y^2 + z^2 = 25$

Select the correct answer.

a. 500π
b. $2{,}500\pi$
c. 500
d. 7,500
e. $7{,}500\pi$

18. Use a computer algebra system to compute the flux of **F** across *S.*

$\mathbf{F}(x, y, z) = \sin x \cos^2 y\mathbf{i} + \sin^3 z \cos^4 z\mathbf{j} + \sin^5 z \cos^6 x\mathbf{k}$

S is the surface of the cube cut from the first octant by the planes $x = \frac{\pi}{2},\ \ y = \frac{\pi}{2},\ \ z = \frac{\pi}{2}$.

Select the correct answer.

a. 0.67
b. 2
c. 4.01
d. 1
e. 3

19. Which of the equations below is an equation of a cone?

a. $r(x,\ \theta) = \left\langle x,\ \cos 7\theta,\ \sin 7\theta \right\rangle$

b. $r(x,\ \theta) = \left\langle x,\ \cos 5\theta,\ \sin 5\theta \right\rangle$

20. Use the Divergence Theorem to calculate the surface integral.

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

$\mathbf{F}(x, y, z) = 4x\mathbf{i} + 5xy\mathbf{j} + 3xz\mathbf{k}$

S is the surface of the box bounded by the planes $x = 0$, $x = 5$, $y = 0$, $y = 3$, $z = 0$, $z = 4$.

Select the correct answer.

a. 2,640 b. 1,440 c. -960 d. 120 e. 140