

2-5 Study Guide and Intervention

Postulates and Paragraph Proofs

Points, Lines, and Planes In geometry, a **postulate** is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate: Through any two points, there is exactly one line.
Postulate: Through any three points not on the same line, there is exactly one plane.
Postulate: A line contains at least two points.
Postulate: A plane contains at least three points not on the same line.
Postulate: If two points lie in a plane, then the line containing those points lies in the plane.
Postulate: If two lines intersect, then their intersection is exactly one point.
Postulate: If two planes intersect, then their intersection is a line.

Example

Determine whether each statement is *always*, *sometimes*, or *never* true.

a. There is exactly one plane that contains points A , B , and C .

Sometimes; if A , B , and C are collinear, they are contained in many planes. If they are noncollinear, then they are contained in exactly one plane.

b. Points E and F are contained in exactly one line.

Always; the first postulate states that there is exactly one line through any two points.

c. Two lines intersect in two distinct points M and N .

Never; the intersection of two lines is one point.

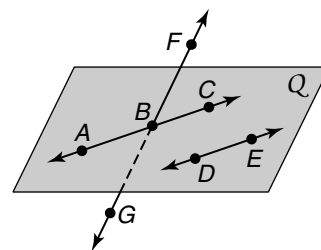
Exercises

Use postulates to determine whether each statement is *always*, *sometimes*, or *never* true.

- A line contains exactly one point.
- Noncollinear points R , S , and T are contained in exactly one plane.
- Any two lines ℓ and m intersect.
- If points G and H are contained in plane \mathcal{M} , then \overline{GH} is perpendicular to plane \mathcal{M} .
- Planes \mathcal{R} and \mathcal{S} intersect in point T .
- If points A , B , and C are noncollinear, then segments \overline{AB} , \overline{BC} , and \overline{CA} are contained in exactly one plane.

In the figure, \overline{AC} and \overline{DE} are in plane Q and $\overline{AC} \parallel \overline{DE}$. State the postulate that can be used to show each statement is true.

- Exactly one plane contains points F , B , and E .
- \overline{BE} lies in plane Q .



2-5 Study Guide and Intervention *(continued)*

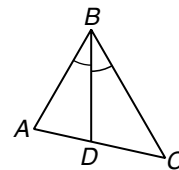
Postulates and Paragraph Proofs

Paragraph Proofs A statement that can be proved true is called a **theorem**. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

A logical argument that uses deductive reasoning to reach a valid conclusion is called a **proof**. In one type of proof, a **paragraph proof**, you write a paragraph to explain why a statement is true.

Example In $\triangle ABC$, \overline{BD} is an angle bisector. Write a paragraph proof to show that $\angle ABD \cong \angle CBD$.

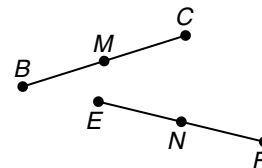
By definition, an angle bisector divides an angle into two congruent angles. Since \overline{BD} is an angle bisector, $\angle ABC$ is divided into two congruent angles. Thus, $\angle ABD \cong \angle CBD$.



Exercises

1. Given that $\angle A \cong \angle D$ and $\angle D \cong \angle E$, write a paragraph proof to show that $\angle A \cong \angle E$.

2. It is given that $\overline{BC} \cong \overline{EF}$, M is the midpoint of \overline{BC} , and N is the midpoint of \overline{EF} . Write a paragraph proof to show that $BM = EN$.



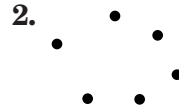
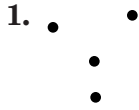
3. Given that S is the midpoint of \overline{QP} , T is the midpoint of \overline{PR} , and P is the midpoint of \overline{ST} , write a paragraph proof to show that $QS = TR$.



2-5 Skills Practice

Postulates and Paragraph Proofs

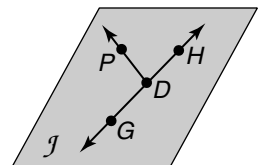
Determine the number of line segments that can be drawn connecting each pair of points.



Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

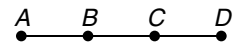
3. Three collinear points determine a plane.
4. Two points A and B determine a line.
5. A plane contains at least three lines.

In the figure, \overleftrightarrow{DG} and \overleftrightarrow{DP} lie in plane J and H lies on \overleftrightarrow{DG} . State the postulate that can be used to show each statement is true.



6. G and P are collinear.
7. Points D , H , and P are coplanar.

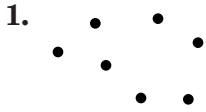
8. **PROOF** In the figure at the right, point B is the midpoint of \overline{AC} and point C is the midpoint of \overline{BD} . Write a paragraph proof to prove that $AB = CD$.



2-5 Practice

Postulates and Paragraph Proofs

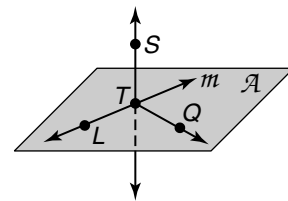
Determine the number of line segments that can be drawn connecting each pair of points.



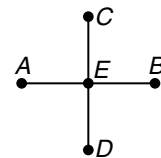
Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

- The intersection of two planes contains at least two points.
- If three planes have a point in common, then they have a whole line in common.

In the figure, line m and \overline{TQ} lie in plane \mathcal{A} . State the postulate that can be used to show that each statement is true.



- L , T , and line m lie in the same plane.
- Line m and \overline{ST} intersect at T .
- In the figure, E is the midpoint of \overline{AB} and \overline{CD} , and $AB = CD$. Write a paragraph proof to prove that $\overline{AE} \cong \overline{ED}$.



- LOGIC** Points A , B , and C are not collinear. Points B , C , and D are not collinear. Points A , B , C , and D are not coplanar. Describe two planes that intersect in line BC .

2-5

Reading to Learn Mathematics***Postulates and Paragraph Proofs*****Pre-Activity** How are postulates used by the founding fathers of the United States?

Read the introduction to Lesson 2-5 at the top of page 89 in your textbook.

Postulates are often described as statements that are so basic and so clearly correct that people will be willing to accept them as true without asking for evidence or proof. Give a statement about numbers that you think most people would accept as true without evidence.

Reading the Lesson

- Determine whether each of the following is a *correct* or *incorrect* statement of a geometric postulate. If the statement is incorrect, replace the underlined words to make the statement correct.
 - A plane contains at least two points that do not lie on the same line.
 - If two planes intersect, then the intersection is a line.
 - Through any four points not on the same line, there is exactly one plane.
 - A line contains at least one point.
 - If two lines are parallel, then their intersection is exactly one point.
 - Through any two points, there is at most one line.
- Determine whether each statement is *always*, *sometimes*, or *never* true. If the statement is not always true, explain why.
 - If two planes intersect, their intersection is a line.
 - The midpoint of a segment divides the segment into two congruent segments.
 - There is exactly one plane that contains three collinear points.

 - If two lines intersect, their intersection is one point.
- Use the walls, floor, and ceiling of your classroom to describe a model for each of the following geometric situations.
 - two planes that intersect in a line

 - two planes that do not intersect

 - three planes that intersect in a point

Helping You Remember

- A good way to remember a new mathematical term is to relate it to a word you already know. Explain how the idea of a mathematical *theorem* is related to the idea of a scientific *theory*.

2-5 Enrichment

Logic Problems

The following problems can be solved by eliminating possibilities. It may be helpful to use charts such as the one shown in the first problem. Mark an X in the chart to eliminate a possible answer.

Solve each problem.

1. Nancy, Olivia, Mario, and Kenji each have one piece of fruit in their school lunch. They have a peach, an orange, a banana, and an apple. Mario does not have a peach or a banana. Olivia and Mario just came from class with the student who has an apple. Kenji and Nancy are sitting next to the student who has a banana. Nancy does not have a peach. Which student has each piece of fruit?

	Nancy	Olivia	Mario	Kenji
Peach				
Orange				
Banana				
Apple				

2. Victor, Leon, Kasha, and Sheri each play one instrument. They play the viola, clarinet, trumpet, and flute. Sheri does not play the flute. Kasha lives near the student who plays flute and the one who plays trumpet. Leon does not play a brass or wind instrument. Which student plays each instrument?

3. Mr. Guthrie, Mrs. Hakoi, Mr. Mirza, and Mrs. Riva have jobs of doctor, accountant, teacher, and office manager. Mr. Mirza lives near the doctor and the teacher. Mrs. Riva is not the doctor or the office manager. Mrs. Hakoi is not the accountant or the office manager. Mr. Guthrie went to lunch with the doctor. Mrs. Riva's son is a high school student and is only seven years younger than his algebra teacher. Which person has each occupation?

4. Yvette, Lana, Boris, and Scott each have a dog. The breeds are collie, beagle, poodle, and terrier. Yvette and Boris walked to the library with the student who has a collie. Boris does not have a poodle or terrier. Scott does not have a collie. Yvette is in math class with the student who has a terrier. Which student has each breed of dog?