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Study Guide and Intervention

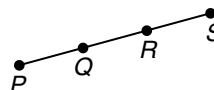
Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate	The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero and B corresponds to a positive real number.
Segment Addition Postulate	B is between A and C if and only if $AB + BC = AC$.

Example

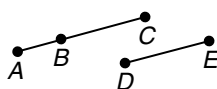
Write a two-column proof.

Given: Q is the midpoint of \overline{PR} . R is the midpoint of \overline{QS} .**Prove:** $PR = QS$ 

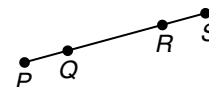
Statements	Reasons
1. Q is the midpoint of \overline{PR} .	1. Given
2. $PQ = QR$	2. Definition of midpoint
3. R is the midpoint of \overline{QS} .	3. Given
4. $QR = RS$	4. Definition of midpoint
5. $PQ + QR = QR + RS$	5. Addition Property
6. $PQ + QR = PR$, $QR + RS = QS$	6. Segment Addition Postulate
7. $PR = QS$	7. Substitution

Exercises

Complete each proof.

1. **Given:** $BC = DE$ **Prove:** $AB + DE = AC$ 

Statements	Reasons
a. $BC = DE$	a. _____
b. _____	b. Seg. Add. Post.
c. $AB + DE = AC$	c. _____

2. **Given:** Q is between P and R , R is between Q and S , $PR = QS$.**Prove:** $PQ = RS$ 

Statements	Reasons
a. Q is between P and R .	a. Given
b. $PQ + QR = PR$	b. _____
c. R is between Q and S .	c. _____
d. _____	d. Seg. Add. Post.
e. $PR = QS$	e. _____
f. $PQ + QR = QR + RS$	f. _____
g. $PQ + QR - QR = QR + RS - QR$	g. _____
h. _____	h. Substitution

2-7 Study Guide and Intervention *(continued)*

Proving Segment Relationships

Segment Congruence Three properties of algebra—the Reflexive, Symmetric, and Transitive Properties of Equality—have counterparts as properties of geometry. These properties can be proved as a theorem. As with other theorems, the properties can then be used to prove relationships among segments.

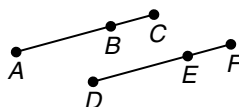
Segment Congruence Theorem	Congruence of segments is reflexive, symmetric, and transitive.
Reflexive Property	$\overline{AB} \cong \overline{AB}$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example

Write a two-column proof.

Given: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$

Prove: $\overline{AC} \cong \overline{DF}$



Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $AB = DE$	2. Definition of congruence of segments
3. $\overline{BC} \cong \overline{EF}$	3. Given
4. $BC = EF$	4. Definition of congruence of segments
5. $AB + BC = DE + EF$	5. Addition Property
6. $AB + BC = AC$, $DE + EF = DF$	6. Segment Addition Postulate
7. $AC = DF$	7. Substitution
8. $\overline{AC} \cong \overline{DF}$	8. Definition of congruence of segments

Exercises

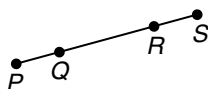
Justify each statement with a property of congruence.

- If $\overline{DE} \cong \overline{GH}$, then $\overline{GH} \cong \overline{DE}$.
- If $\overline{AB} \cong \overline{RS}$ and $\overline{RS} \cong \overline{WY}$, then $\overline{AB} \cong \overline{WY}$.
- $\overline{RS} \cong \overline{RS}$

- Complete the proof.

Given: $\overline{PR} \cong \overline{QS}$

Prove: $\overline{PQ} \cong \overline{RS}$



Statements	Reasons
a. $\overline{PR} \cong \overline{QS}$	a. _____
b. $PR = QS$	b. _____
c. $PQ + QR = PR$	c. _____
d. _____	d. Segment Addition Postulate
e. $PQ + QR = QR + RS$	e. _____
f. _____	f. Subtraction Property
g. _____	g. Definition of congruence of segments

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Skills Practice

Proving Segment Relationships

Justify each statement with a property of equality, a property of congruence, or a postulate.

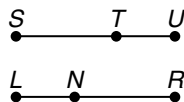
- $QA = QA$
- If $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CE}$, then $\overline{AB} \cong \overline{CE}$.
- If Q is between P and R , then $PR = PQ + QR$.
- If $AB + BC = EF + FG$ and $AB + BC = AC$, then $EF + FG = AC$.

Complete each proof.

5. Given: $\overline{SU} \cong \overline{LR}$
 $\overline{TU} \cong \overline{LN}$

Prove: $\overline{ST} \cong \overline{NR}$

Proof:



Statements

Reasons

a. $\overline{SU} \cong \overline{LR}, \overline{TU} \cong \overline{LN}$

a. _____

b. _____

b. Definition of \cong segments

c. $SU = ST + TU$

c. _____

$LR = LN + NR$

d. $ST + TU = LN + NR$

d. _____

e. $ST + LN = LN + NR$

e. _____

f. $ST + LN - LN = LN + NR - LN$

f. _____

g. _____

g. Substitution Property

h. $\overline{ST} \cong \overline{NR}$

h. _____

6. Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{CD} \cong \overline{AB}$

Proof:

Statements

Reasons

a. _____

a. Given

b. $AB = CD$

b. _____

c. $CD = AB$

c. _____

d. _____

d. Definition of \cong segments

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Practice

Proving Segment Relationships

Complete the following proof.

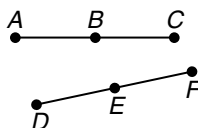
1. **Given:** $\overline{AB} \cong \overline{DE}$

B is the midpoint of \overline{AC} .

E is the midpoint of \overline{DF} .

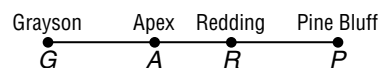
Prove: $\overline{BC} \cong \overline{EF}$

Proof:



Statements	Reasons
a. _____	a. Given
b. $AB = DE$	b. _____
c. _____	c. Definition of Midpoint
d. $AC = AB + BC$ $DF = DE + EF$	d. _____
e. $AB + BC = DE + EF$	e. _____
f. $AB + BC = AB + EF$	f. _____
g. $AB + BC - AB = AB + EF - AB$	g. Subtraction Property
h. $BC = EF$	h. _____
i. _____	i. _____

2. **TRAVEL** Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.



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Reading to Learn Mathematics

Proving Segment Relationships

Pre-Activity How can segment relationships be used for travel?

Read the introduction to Lesson 2-7 at the top of page 101 in your textbook.

- What is the total distance that the plane will fly to get from San Diego to Dallas?
- Before leaving home, a passenger used a road atlas to determine that the distance between San Diego and Dallas is about 1350 miles. Why is the flying distance greater than that?

Reading the Lesson

1. If E is between Y and S , which of the following statements are *always* true?

A. $YS + ES = YE$

B. $YS - ES = YE$

C. $YE > ES$

D. $YE \cdot ES = YS$

E. $SE + EY = SY$

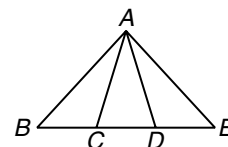
F. E is the midpoint of \overline{YS} .

2. Give the reason for each statement in the following two-column proof.

Given: C is the midpoint of \overline{BD} .

D is the midpoint of \overline{CE} .

Prove: $\overline{BD} \cong \overline{CE}$



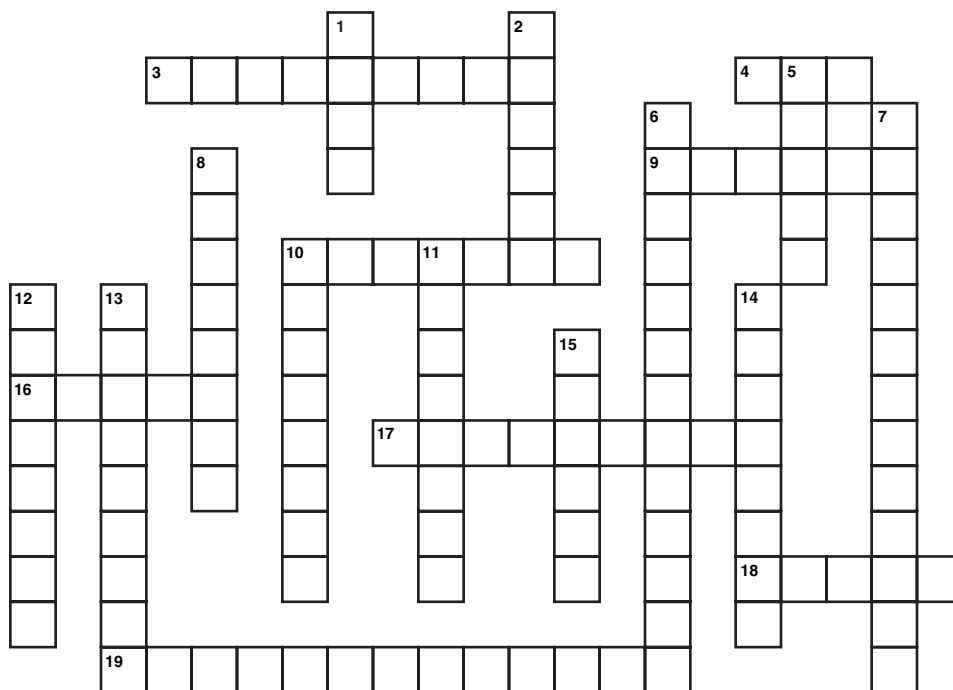
Statements	Reasons
1. C is the midpoint of \overline{BD} .	1. _____
2. $BC = CD$	2. _____
3. D is the midpoint of \overline{CE} .	3. _____
4. $CD = DE$	4. _____
5. $BC = DE$	5. _____
6. $BC + CD = CD + DE$	6. _____
7. $BC + CD = BD$ $CD + DE = CE$	7. _____
8. $BD = CE$	8. _____
9. $\overline{BD} \cong \overline{CE}$	9. _____

Helping You Remember

3. One way to keep the names of related postulates straight in your mind is to associate something in the name of the postulate with the content of the postulate. How can you use this idea to distinguish between the Ruler Postulate and the Segment Addition Postulate?

2-7 Enrichment

Geometry Crossword Puzzle



ACROSS

3. Points on the same line are _____.
4. A point on a line and all points of the line to one side of it.
9. An angle whose measure is greater than 90.
10. Two endpoints and all points between them.
16. A flat figure with no thickness that extends indefinitely in all directions.
17. Segments of equal length are _____ segments.
18. Two noncollinear rays with a common endpoint.
19. If $m\angle A + m\angle B = 180$, then $\angle A$ and $\angle B$ are _____ angles.

DOWN

1. The set of all points collinear to two points is a _____.
2. The point where the x - and y -axis meet.
5. An angle whose measure is less than 90.
6. If $m\angle A + m\angle D = 90$, then $\angle A$ and $\angle D$ are _____ angles.
7. Lines that meet at a 90° angle are _____.
8. Two angles with a common side but no common interior points are _____.
10. An "angle" formed by opposite rays is a _____ angle.
11. The middle point of a line segment.
12. Points that lie in the same plane are _____.
13. The four parts of a coordinate plane.
14. Two nonadjacent angles formed by two intersecting lines are _____ angles.
15. In angle ABC , point B is the _____.