### Proportions

NAME

**Write Ratios** A **ratio** is a comparison of two quantities. The ratio *a* to *b*, where *b* is not zero, can be written as  $\frac{a}{b}$  or *a*:*b*. The ratio of two quantities is sometimes called a **scale factor**. For a scale factor, the units for each quantity are the same.

**Example 1** In 2002, the Chicago Cubs baseball team won 67 games out of 162. Write a ratio for the number of games won to the total number of games played. To find the ratio, divide the number of games won by the total number of games played. The result is  $\frac{67}{162}$ , which is about 0.41. The Chicago Cubs won about 41% of their games in 2002.

# **Example 2** A doll house that is 15 inches tall is a scale model of a real house with a height of 20 feet. What is the ratio of the height of the doll house to the height of the real house?

To start, convert the height of the real house to inches.

20 feet  $\times$  12 inches per foot = 240 inches

To find the ratio or scale factor of the heights, divide the height of the doll house by the height of the real house. The ratio is 15 inches: 240 inches or 1:16. The height of the doll house is  $\frac{1}{16}$  the height of the real house.

#### Exercises

- 1. In the 2002 Major League baseball season, Sammy Sosa hit 49 home runs and was at bat 556 times. Find the ratio of home runs to the number of times he was at bat.
- **2.** There are 182 girls in the sophomore class of 305 students. Find the ratio of girls to total students.
- **3.** The length of a rectangle is 8 inches and its width is 5 inches. Find the ratio of length to width.
- **4.** The sides of a triangle are 3 inches, 4 inches, and 5 inches. Find the scale factor between the longest and the shortest sides.
- **5.** The length of a model train is 18 inches. It is a scale model of a train that is 48 feet long. Find the scale factor.

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## Study Guide and Intervention (continued)

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#### **Proportions**

**Use Properties of Proportions** A statement that two ratios are equal is called a **proportion**. In the proportion  $\frac{a}{b} = \frac{c}{d}$ , where *b* and *d* are not zero, the values *a* and *d* are the **extremes** and the values *b* and *c* are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so ad = bc.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \cdot d = b \cdot c$$

$$\uparrow \qquad \uparrow$$
extremes means

Example 1	Solve $\frac{9}{16} = \frac{27}{7}$ .			
$\frac{9}{16} = \frac{27}{x}$	10 x			
$9 \cdot x = 16 \cdot 27$	Cross products			
9x = 432	Multiply.			
x = 48	Divide each side by 9.			

# **Example 2** A room is 49 centimeters by 28 centimeters on a scale drawing of a house. For the actual room, the larger dimension is 14 feet. Find the shorter dimension of the actual room.

If x is the room's shorter dimension, then

$\frac{28}{49} = \frac{x}{14}$	shorter dimension longer dimension
49x = 392	Cross products
x = 8	Divide each side by 49

The shorter side of the room is 8 feet.

#### Exercises

#### Solve each proportion.

<b>1.</b> $\frac{1}{2} = \frac{28}{x}$	<b>2.</b> $\frac{3}{8} = \frac{y}{24}$	<b>3.</b> $\frac{x+22}{x+2} = \frac{30}{10}$
<b>4.</b> $\frac{3}{18.2} = \frac{9}{y}$	<b>5.</b> $\frac{2x+3}{8} = \frac{5}{4}$	<b>6.</b> $\frac{x+1}{x-1} = \frac{3}{4}$

#### Use a proportion to solve each problem.

7. If 3 cassettes cost \$44.85, find the cost of one cassette.

- **8.** The ratio of the sides of a triangle are 8:15:17. If the perimeter of the triangle is 480 inches, find the length of each side of the triangle.
- **9.** The scale on a map indicates that one inch equals 4 miles. If two towns are 3.5 inches apart on the map, what is the actual distance between the towns?

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### **Proportions**

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- **1. FOOTBALL** A tight end scored 6 touchdowns in 14 games. Find the ratio of touchdowns per game.
- **2. EDUCATION** In a schedule of 6 classes, Marta has 2 elective classes. What is the ratio of elective to non-elective classes in Marta's schedule?
- **3. BIOLOGY** Out of 274 listed species of birds in the United States, 78 species made the endangered list. Find the ratio of endangered species of birds to listed species in the United States.
- **4. ART** An artist in Portland, Oregon, makes bronze sculptures of dogs. The ratio of the height of a sculpture to the actual height of the dog is 2:3. If the height of the sculpture is 14 inches, find the height of the dog.
- **5. SCHOOL** The ratio of male students to female students in the drama club at Campbell High School is 3:4. If the number of male students in the club is 18, what is the number of female students?

#### Solve each proportion.



#### Find the measures of the sides of each triangle.

**12.** The ratio of the measures of the sides of a triangle is 3:5:7, and its perimeter is 450 centimeters.

13. The ratio of the measures of the sides of a triangle is 5:6:9, and its perimeter is 220 meters.

14. The ratio of the measures of the sides of a triangle is 4:6:8, and its perimeter is 126 feet.

15. The ratio of the measures of the sides of a triangle is 5:7:8, and its perimeter is 40 inches.

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## Practice Proportions

- **1. NUTRITION** One ounce of cheddar cheese contains 9 grams of fat. Six of the grams of fat are saturated fats. Find the ratio of saturated fats to total fat in an ounce of cheese.
- **2. FARMING** The ratio of goats to sheep at a university research farm is 4:7. The number of sheep at the farm is 28. What is the number of goats?
- **3. ART** Edward Hopper's oil on canvas painting *Nighthawks* has a length of 60 inches and a width of 30 inches. A print of the original has a length of 2.5 inches. What is the width of the print?

#### Solve each proportion.

<b>4.</b> $\frac{5}{8} = \frac{x}{12}$	<b>5.</b> $\frac{x}{1.12} = \frac{1}{5}$	<b>6.</b> $\frac{6x}{27} = \frac{4}{3}$
<b>7.</b> $\frac{x+2}{3} = \frac{8}{9}$	8. $\frac{3x-5}{4} = \frac{-5}{7}$	9. $\frac{x-2}{4} = \frac{x+4}{2}$

#### Find the measures of the sides of each triangle.

10. The ratio of the measures of the sides of a triangle is 3:4:6, and its perimeter is 104 feet.

- 11. The ratio of the measures of the sides of a triangle is 7:9:12, and its perimeter is 84 inches.
- **12.** The ratio of the measures of the sides of a triangle is 6:7:9, and its perimeter is 77 centimeters.

#### Find the measures of the angles in each triangle.

- **13.** The ratio of the measures of the angles is 4:5:6.
- **14.** The ratio of the measures of the angles is 5:7:8.
- **15. BRIDGES** The span of the Benjamin Franklin suspension bridge in Philadelphia, Pennsylvania, is 1750 feet. A model of the bridge has a span of 42 inches. What is the ratio of the span of the model to the span of the actual Benjamin Franklin Bridge?

Similar Polygons

**Identify Similar Figures** 

NAME

#### **Determine whether the triangles**

#### are similar.

Example 1

Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.

 $\angle C \cong \angle Z$  because they are right angles, and  $\angle B \cong \angle X$ . By the Third Angle Theorem,  $\angle A \cong \angle Y$ .

For the sides,  $\frac{BC}{XZ} = \frac{20}{23}$ ,  $\frac{BA}{XY} = \frac{20\sqrt{2}}{23\sqrt{2}} = \frac{20}{23}$ , and  $\frac{AC}{YZ} = \frac{20}{23}$ .

The side lengths are proportional. So  $\triangle BCA \sim \triangle XZY$ .

#### Example 2

#### Is polygon WXYZ ~ polygon PQRS?

For the sides,  $\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}, \frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}, \frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2},$ and  $\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$ . So corresponding sides are proportional. Also,  $\angle W \cong \angle P$ ,  $\angle X \cong \angle Q$ ,  $\angle Y \cong \angle R$ , and  $\angle Z \cong \angle S$ , so corresponding angles are congruent. We can conclude that polygon  $WXYZ \sim$  polygon PQRS.





#### Exercises

Determine whether each pair of figures is similar. If they are similar, give the ratio of corresponding sides.









NAME

## Study Guide and Intervention (continued) Similar Polygons

**Scale Factors** When two polygons are similar, the ratio of the lengths of corresponding sides is called the scale factor. At the right,  $\triangle ABC \sim \triangle XYZ$ . The scale factor of  $\triangle ABC$  to  $\triangle XYZ$  is 2 and the scale factor of  $\triangle XYZ$  to  $\triangle ABC$  is  $\frac{1}{2}$ .



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DE = 2(BC) or  $2\sqrt{13}$ .

Exercises

Each pair of polygons is similar. Find *x* and *y*.



**5.** In Example 2 above, point D has coordinates (5, 6). Use the Distance Formula to verify the lengths of CD and DE.

Similar Polygons

Determine whether each pair of figures is similar. Justify your answer.





Each pair of polygons is similar. Write a similarity statement, and find x, the measure(s) of the indicated side(s), and the scale factor.







5.  $\overline{WT}$ 



**6.**  $\overline{TS}$  and  $\overline{SP}$ 





Each pair of polygons is similar. Write a similarity statement, and find x, the measure(s) of the indicated side(s), and the scale factor.

**3.**  $\overline{LM}$  and  $\overline{MN}$ 







**5. COORDINATE GEOMETRY** Triangle *ABC* has vertices A(0, 0), B(-4, 0), and C(-2, 4). The coordinates of each vertex are multiplied by 3 to create  $\triangle AEF$ . Show that  $\triangle AEF$  is similar to  $\triangle ABC$ .

**6. INTERIOR DESIGN** Graham used the scale drawing of his living room to decide where to place furniture. Find the dimensions of the living room if the scale in the drawing is 1 inch = 4.5 feet.



Similar Triangles

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Identify Similar Triangles Here are three ways to show that two triangles are similar.

AA Similarity	Two angles of one triangle are congruent to two angles of another triangle.
SSS Similarity	The measures of the corresponding sides of two triangles are proportional.
SAS Similarity	The measures of two sides of one triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.



Example 2 **Determine whether the** triangles are similar. ≥P 4/70°  $\frac{3}{4} = \frac{6}{8}$ , so  $\frac{MN}{QR} = \frac{NP}{RS}$ .

 $m \angle N = m \angle R$ , so  $\angle N \cong \angle R$ .  $\triangle NMP \sim \triangle RQS$  by SAS Similarity.

Exercises

#### Determine whether each pair of triangles is similar. Justify your answer.













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## Study Guide and Intervention (continued)

### Similar Triangles

**Use Similar Triangles** Similar triangles can be used to find measurements.

Example 1	$\triangle ABC \sim \triangle DEF.$
Find <i>x</i> and <i>y</i> .	
$A \xrightarrow{\frac{y}{18\sqrt{3}}}^{B} C$	$D \xrightarrow{18}_{x} F^{9}$
$\frac{AC}{DF} = \frac{BC}{EF}$ $\frac{18\sqrt{3}}{r} = \frac{18}{9}$	$\frac{AB}{DE} = \frac{BC}{EF}$ $\frac{y}{18} = \frac{18}{9}$
$18x = 9(18\sqrt{3})$ $x = 9\sqrt{3}$	9y = 324 $y = 36$

**Example 2** A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?



The sun's rays form similar triangles. Using x for the height of the pole,  $\frac{6}{x} = \frac{1.5}{7}$ , so 1.5x = 42 and x = 28. The flagpole is 28 feet tall.

#### Exercises

Each pair of triangles is similar. Find *x* and *y*.



7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post's shadow to the nearest hundredth?

Similar Triangles

Determine whether each pair of triangles is similar. Justify your answer.



ALGEBRA Identify the similar triangles, and find x and the measures of the indicated sides.





**7.**  $\overline{EH}$  and  $\overline{EF}$ 



**8.**  $\overline{UT}$  and  $\overline{RT}$ 14 6 V x-6



ALGEBRA Identify the similar triangles, and find x and the measures of the indicated sides.



#### Use the given information to find each measure.





**6.** If  $\overline{EF} \parallel \overline{HI}$ , EF = 3, EG = x + 1, HI = 4, and HG = x + 3, find EG and HG.



#### **INDIRECT MEASUREMENT** For Exercises 7 and 8, use the following information.

A lighthouse casts a 128-foot shadow. A nearby lamppost that measures 5 feet 3 inches casts an 8-foot shadow.

- 7. Write a proportion that can be used to determine the height of the lighthouse.
- 8. What is the height of the lighthouse?

### Parallel Lines and Proportional Parts

**Proportional Parts of Triangles** In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. The converse is also true.



If *X* and *Y* are the midpoints of  $\overline{RT}$  and  $\overline{ST}$ , then  $\overline{XY}$  is a **midsegment** of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

If  $\overrightarrow{XY} \parallel \overrightarrow{RS}$ , then  $\frac{RX}{XT} = \frac{SY}{YT}$ . If  $\frac{RX}{XT} = \frac{SY}{YT}$ , then  $\overleftarrow{XY} \parallel \overleftarrow{RS}$ . If  $\overline{XY}$  is a midsegment, then  $\overleftarrow{XY} \parallel \overleftarrow{RS}$  and  $XY = \frac{1}{2}RS$ .



Example 2 A triangle has vertices D(3, 6), E(-3, -2), and F(7, -2).Midsegment  $\overline{GH}$  is parallel to  $\overline{EF}$ . Find the length of  $\overline{GH}$ .

 $\overline{GH}$  is a midsegment, so its length is onehalf that of  $\overline{EF}$ . Points *E* and *F* have the same y-coordinate, so EF = 7 - (-3) = 10. The length of midsegment  $\overline{GH}$  is 5.

#### Exercises





**7.** In Example 2, find the slope of  $\overline{EF}$  and show that  $\overline{EF} \parallel \overline{GH}$ .

Lesson 6-4

NAME

## Study Guide and Intervention (continued)

Parallel Lines and Proportional Parts

**Divide Segments Proportionally** When three or more parallel lines cut two transversals, they separate the transversals into proportional parts. If the ratio of the parts is 1, then the parallel lines separate the transversals into congruent parts.



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 $\begin{array}{ll} \text{If } \ell_1 \parallel \ell_2 \parallel \ell_3, \qquad \text{If } \ell_4 \parallel \ell_5 \parallel \ell_6 \text{ and} \\ \text{then } \frac{a}{b} = \frac{c}{d}. \qquad \frac{u}{v} = 1, \text{ then } \frac{w}{x} = 1. \end{array}$ 

## Example Refer to lines $\ell_1$ , $\ell_2$ , and $\ell_3$ above. If a = 3, b = 8, and c = 5, find d. $\ell_1 \parallel \ell_2 \parallel \ell_3$ so $\frac{3}{8} = \frac{5}{d}$ . Then 3d = 40 and $d = 13\frac{1}{3}$ .

#### Exercises

#### Find x and y.













Parallel Lines and Proportional Parts

**1.** If JK = 7, KH = 21, and JL = 6, find *LI*.



**2.** Find *x* and *TV* if RU = 8, US = 14, TV = x - 1 and VS = 17.5.



#### Determine whether $\overline{BC} \parallel \overline{DE}$ .

**3.** AD = 15, DB = 12, AE = 10, and EC = 8

**4.** BD = 9, BA = 27, and CE is one third of EA

**5.** AE = 30, AC = 45, and AD is twice DB

#### **COORDINATE GEOMETRY** For Exercises 6-8, use the following information.

Triangle *ABC* has vertices A(-5, 2), B(1, 8), and C(4, 2). Point D is the midpoint of  $\overline{AB}$  and *E* is the midpoint of  $\overline{AC}$ .

- **6.** Identify the coordinates of D and E.
- **7.** Show that  $\overline{BC}$  is parallel to  $\overline{DE}$ .
- 8. Show that  $DE = \frac{1}{2}BC$ .





**10.** Find *x* and *y*.







Homework 6.2 NAME

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## **Practice**

## Parallel Lines and Proportional Parts

**1.** If AD = 24, DB = 27, and EB = 18, find *CE*.



Determine whether  $\overline{JK} \parallel \overline{NM}$ .

**3.** JN = 18, JL = 30, KM = 21, and ML = 35

**4.**  $KM = 24, KL = 44, \text{ and } NL = \frac{5}{6}JN$ 

#### **COORDINATE GEOMETRY** For Exercises 5 and 6, use the following information.

Triangle *EFG* has vertices E(-4, -1), F(2, 5), and G(2, -1). Point K is the midpoint of  $\overline{EG}$  and *H* is the midpoint of  $\overline{FG}$ .

**5.** Show that  $\overline{EF}$  is parallel to  $\overline{KH}$ .







				y	F	
					Н	
		/				
_	$\angle$		0			
						x
Ε		K			G	

**6.** Show that 
$$KH = \frac{1}{2}EF$$
.

**7.** Find *x* and *y*.



**8.** Find *x* and *y*.



9. MAPS The distance from Wilmington to Ash Grove along Kendall is 820 feet and along Magnolia, 660 feet. If the distance between Beech and Ash Grove along Magnolia is 280 feet, what is the distance between the two streets along Kendall?



## **Study Guide and Intervention** Parts of Similar Triangles

**Perimeters** If two triangles are similar, their perimeters have the same proportion as the corresponding sides.

If  $\triangle ABC \sim \triangle RST$ , then  $\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}.$ 



#### Example Use the diagram above with $\triangle ABC \sim \triangle RST$ . If AB = 24 and RS = 15, find the ratio of their perimeters.

Since  $\triangle ABC \sim \triangle RST$ , the ratio of the perimeters of  $\triangle ABC$  and  $\triangle RST$  is the same as the ratio of corresponding sides.

Therefore  $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle RST} = \frac{24}{15}$  $=\frac{8}{5}$ 

#### Exercises

Each pair of triangles is similar. Find the perimeter of the indicated triangle.

**1.**  $\triangle XYZ$ 





**2.**  $\triangle BDE$ 









**5.** △*ABC* 



**6.**  $\triangle RST$ 





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## Study Guide and Intervention (continued) Parts of Similar Triangles

Special Segments of Similar Triangles When two triangles are similar,

corresponding altitudes, angle bisectors, and medians are proportional to the corresponding sides. Also, in any triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides of the triangle.



Find x for each pair of similar triangles.



### Parts of Similar Triangles

#### Find the perimeter of the given triangle.

**1.**  $\triangle JKL$ , if  $\triangle JKL \sim \triangle RST$ , RS = 14, ST = 12, TR = 10, and LJ = 14





**3.**  $\triangle PQR$ , if  $\triangle PQR \sim \triangle LMN$ , LM = 16, MN = 14, NL = 27, and RP = 18



**4.**  $\triangle$ *KLM*, if  $\triangle$ *KLM*  $\sim \triangle$ *FGH*, *FG* = 30, GH = 38, HF = 38, and KL = 24



#### Use the given information to find each measure.

**5.** Find *FG* if  $\triangle RST \sim \triangle EFG$ ,  $\overline{SH}$  is an altitude of  $\triangle RST$ ,  $\overline{FJ}$  is an altitude of  $\triangle EFG$ , ST = 6, SH = 5, and FJ = 7.



**6.** Find *MN* if  $\triangle ABC \sim \triangle MNP$ ,  $\overline{AD}$  is an altitude of  $\triangle ABC$ ,  $\overline{MQ}$  is an altitude of  $\triangle MNP, AB = 24, AD = 14, \text{ and } MQ = 10.5.$ 



Find *x*.





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## **Practice** Parts of Similar Triangles

#### Find the perimeter of the given triangle.

**1.**  $\triangle DEF$ , if  $\triangle ABC \sim \triangle DEF$ , AB = 36, BC = 20, CA = 40, and DE = 35





#### Use the given information to find each measure.

**3.** Find *PR* if  $\triangle JKL \sim \triangle NPR$ ,  $\overline{KM}$  is an altitude of  $\triangle JKL$ ,  $\overline{PT}$  is an altitude of  $\triangle NPR$ , KL = 28, KM = 18, and PT = 15.75.







#### Find *x*.





#### **PHOTOGRAPHY** For Exercises 7 and 8, use the following information.

Francine has a camera in which the distance from the lens to the film is 24 millimeters.

- **7.** If Francine takes a full-length photograph of her friend from a distance of 3 meters and the height of her friend is 140 centimeters, what will be the height of the image on the film? (*Hint*: Convert to the same unit of measure.)
- **8.** Suppose the height of the image on the film of her friend is 15 millimeters. If Francine took a full-length shot, what was the distance between the camera and her friend?