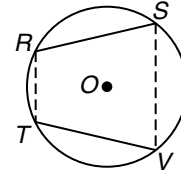


## 10-3 Study Guide and Intervention

### Arcs and Chords

**Arcs and Chords** Points on a circle determine both chords and arcs. Several properties are related to points on a circle.

- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.



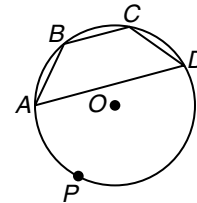
$\overline{RS} \cong \overline{TV}$  if and only if  $\overline{RS} \cong \overline{TV}$ .  
 $RSVT$  is inscribed in  $\odot O$ .  
 $\odot O$  is circumscribed about  $RSVT$ .

#### Example

Trapezoid  $ABCD$  is inscribed in  $\odot O$ .

If  $\overline{AB} \cong \overline{BC} \cong \overline{CD}$  and  $m\widehat{BC} = 50$ , what is  $m\widehat{APD}$ ?

Chords  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CD}$  are congruent, so  $\widehat{AB}$ ,  $\widehat{BC}$ , and  $\widehat{CD}$  are congruent.  $m\widehat{BC} = 50$ , so  $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} = 50 + 50 + 50 = 150$ . Then  $m\widehat{APD} = 360 - 150$  or 210.



#### Exercises

Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.

1. hexagon

2. pentagon

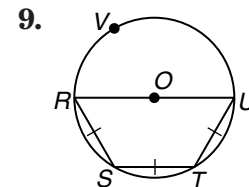
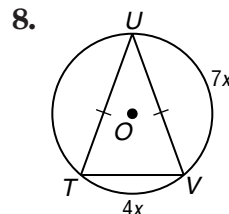
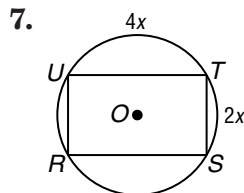
3. triangle

4. square

5. octagon

6. 36-gon

Determine the measure of each arc of the circle circumscribed about the polygon.

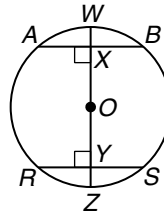


## 10-3 Study Guide and Intervention *(continued)*

### Arcs and Chords

#### Diameters and Chords

- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



If  $\overline{WZ} \perp \overline{AB}$ , then  $\overline{AX} \cong \overline{XB}$  and  $\widehat{AW} \cong \widehat{WB}$ .

If  $OX = OY$ , then  $\overline{AB} \cong \overline{RS}$ .

If  $\overline{AB} \cong \overline{RS}$ , then  $\overline{AB}$  and  $\overline{RS}$  are equidistant from point  $O$ .

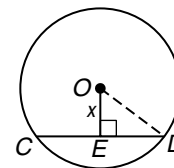
**Example** In  $\odot O$ ,  $\overline{CD} \perp \overline{OE}$ ,  $OD = 15$ , and  $CD = 24$ . Find  $x$ .

A diameter or radius perpendicular to a chord bisects the chord, so  $ED$  is half of  $CD$ .

$$\begin{aligned} ED &= \frac{1}{2}(24) \\ &= 12 \end{aligned}$$

Use the Pythagorean Theorem to find  $x$  in  $\triangle OED$ .

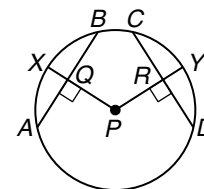
$(OE)^2 + (ED)^2 = (OD)^2$	Pythagorean Theorem
$x^2 + 12^2 = 15^2$	Substitution
$x^2 + 144 = 225$	Multiply.
$x^2 = 81$	Subtract 144 from each side.
$x = 9$	Take the square root of each side.



#### Exercises

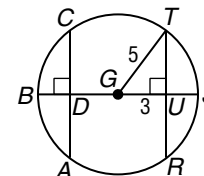
In  $\odot P$ ,  $CD = 24$  and  $m\widehat{CY} = 45$ . Find each measure.

- |                    |                    |                    |
|--------------------|--------------------|--------------------|
| 1. $AQ$            | 2. $RC$            | 3. $QB$            |
| 4. $AB$            | 5. $m\widehat{DY}$ | 6. $m\widehat{AB}$ |
| 7. $m\widehat{AX}$ | 8. $m\widehat{XB}$ | 9. $m\widehat{CD}$ |



In  $\odot G$ ,  $DG = GU$  and  $AC = RT$ . Find each measure.

- |          |          |                     |
|----------|----------|---------------------|
| 10. $TU$ | 11. $TR$ | 12. $m\widehat{TS}$ |
| 13. $CD$ | 14. $GD$ | 15. $m\widehat{AB}$ |



16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius.

## 10-3 Skills Practice

### Arcs and Chords

In  $\odot H$ ,  $m\widehat{RS} = 82$ ,  $m\widehat{TU} = 82$ ,  $RS = 46$ , and  $\overline{TU} \cong \overline{RS}$ . Find each measure.

1.  $TU$

2.  $TK$

3.  $MS$

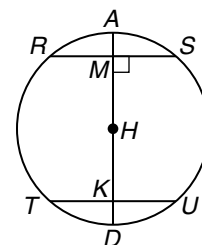
4.  $m\angle HKU$

5.  $m\widehat{AS}$

6.  $m\widehat{AR}$

7.  $m\widehat{TD}$

8.  $m\widehat{DU}$



The radius of  $\odot Y$  is 34,  $AB = 60$ , and  $m\widehat{AC} = 71$ . Find each measure.

9.  $m\widehat{BC}$

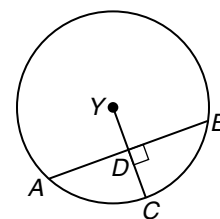
10.  $m\widehat{AB}$

11.  $AD$

12.  $BD$

13.  $YD$

14.  $DC$



In  $\odot X$ ,  $LX = MX$ ,  $XY = 58$ , and  $VW = 84$ . Find each measure.

15.  $YZ$

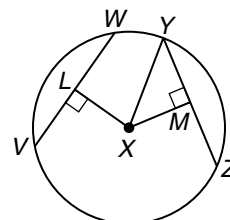
16.  $YM$

17.  $MX$

18.  $MZ$

19.  $LV$

20.  $LX$



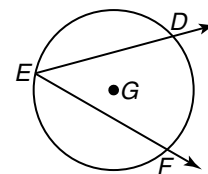
## 10-4 Study Guide and Intervention

### Inscribed Angles

**Inscribed Angles** An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In  $\odot G$ , inscribed  $\angle DEF$  intercepts  $\widehat{DF}$ .

#### Inscribed Angle Theorem

If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.



$$m\angle DEF = \frac{1}{2}m\widehat{DF}$$

#### Example

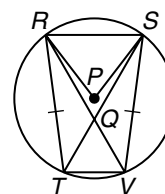
In  $\odot G$  above,  $m\widehat{DF} = 90$ . Find  $m\angle DEF$ .

$\angle DEF$  is an inscribed angle so its measure is half of the intercepted arc.

$$\begin{aligned} m\angle DEF &= \frac{1}{2}m\widehat{DF} \\ &= \frac{1}{2}(90) \text{ or } 45 \end{aligned}$$

#### Exercises

Use  $\odot P$  for Exercises 1–10. In  $\odot P$ ,  $\overline{RS} \parallel \overline{TV}$  and  $\overline{RT} \cong \overline{SV}$ .



- Name the intercepted arc for  $\angle RTS$ .
- Name an inscribed angle that intercepts  $\widehat{SV}$ .

In  $\odot P$ ,  $m\widehat{SV} = 120$  and  $m\angle RPS = 76$ . Find each measure.

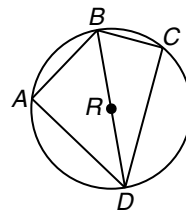
- $m\angle PRS$
- $m\widehat{RSV}$
- $m\widehat{RT}$
- $m\angle RVT$
- $m\angle QRS$
- $m\angle STV$
- $m\widehat{TV}$
- $m\angle SVT$

## 10-4 Study Guide and Intervention *(continued)*

### Inscribed Angles

**Angles of Inscribed Polygons** An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.

- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



If  $\widehat{BCD}$  is a semicircle, then  $m\angle BCD = 90$ .

For inscribed quadrilateral  $ABCD$ ,  
 $m\angle A + m\angle C = 180$  and  
 $m\angle ABC + m\angle ADC = 180$ .

#### Example

In  $\odot R$  above,  $BC = 3$  and  $BD = 5$ . Find each measure.

a.  $m\angle C$

$\angle C$  intercepts a semicircle. Therefore  $\angle C$  is a right angle and  $m\angle C = 90$ .

b.  $CD$

$\triangle BCD$  is a right triangle, so use the Pythagorean Theorem to find  $CD$ .

$$(CD)^2 + (BC)^2 = (BD)^2$$

$$(CD)^2 + 3^2 = 5^2$$

$$(CD)^2 = 25 - 9$$

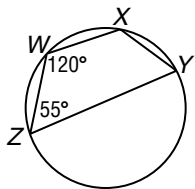
$$(CD)^2 = 16$$

$$CD = 4$$

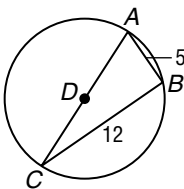
#### Exercises

Find the measure of each angle or segment for each figure.

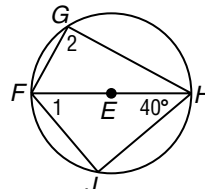
1.  $m\angle X$ ,  $m\angle Y$



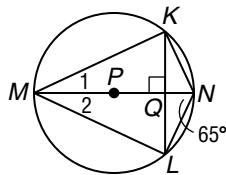
2.  $AD$



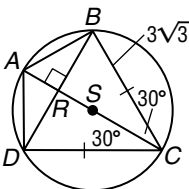
3.  $m\angle 1$ ,  $m\angle 2$



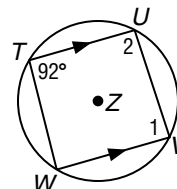
4.  $m\angle 1$ ,  $m\angle 2$



5.  $AB$ ,  $AC$



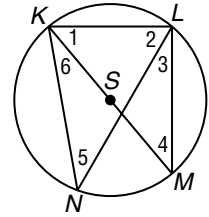
6.  $m\angle 1$ ,  $m\angle 2$



# 10-4 Skills Practice

## Inscribed Angles

In  $\odot S$ ,  $m\widehat{KL} = 80$ ,  $m\widehat{LM} = 100$ , and  $m\widehat{MN} = 60$ . Find the measure of each angle.



1.  $m\angle 1$

2.  $m\angle 2$

3.  $m\angle 3$

4.  $m\angle 4$

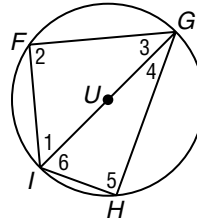
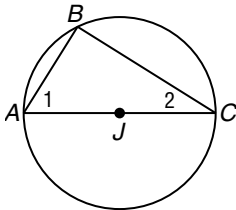
5.  $m\angle 5$

6.  $m\angle 6$

**ALGEBRA** Find the measure of each numbered angle.

7.  $m\angle 1 = 5x - 2$ ,  $m\angle 2 = 2x + 8$

8.  $m\angle 1 = 5x$ ,  $m\angle 3 = 3x + 10$ ,  
 $m\angle 4 = y + 7$ ,  $m\angle 6 = 3y + 11$



Quadrilateral  $RSTU$  is inscribed in  $\odot P$  such that  $m\widehat{STU} = 220$  and  $m\angle S = 95$ . Find each measure.

9.  $m\angle R$

10.  $m\angle T$

11.  $m\angle U$

12.  $m\widehat{SRU}$

13.  $m\widehat{RUT}$

14.  $m\widehat{RST}$

