Learning Curves Are Steeped in Different Relationships

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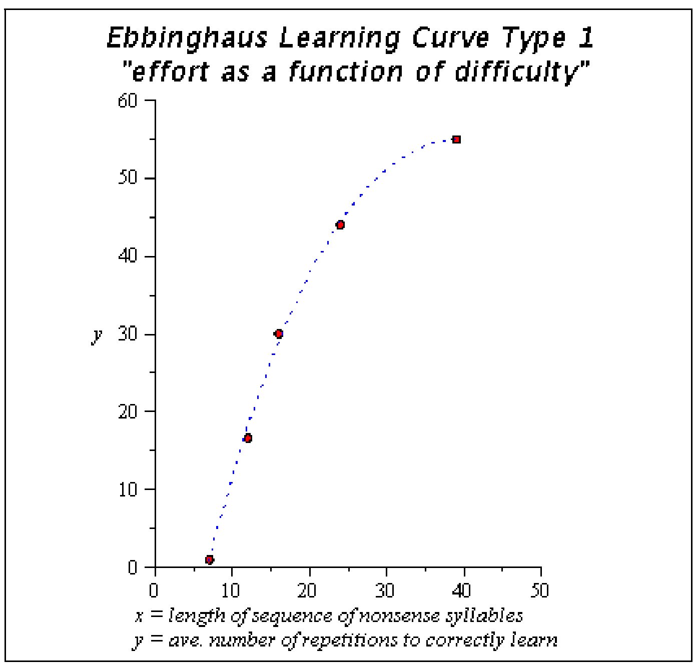
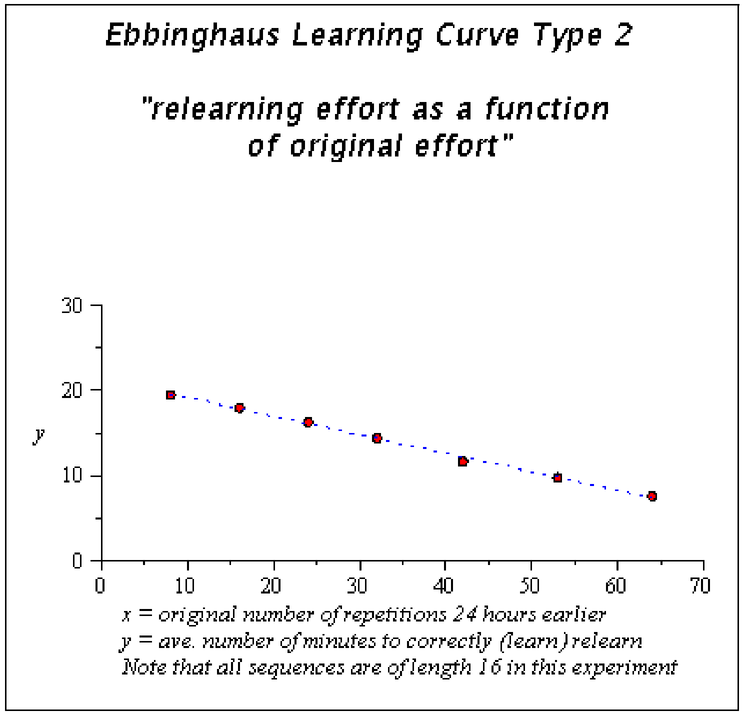
In order to describe the difficulty of learning some particular skill, task, or concept, we often hear someone say that the corresponding learning process has a "steep learning curve." Usually this description is a relative one, and so, we may hear for example that the quadratic formula has a *steep learning curve* compared to simple factoring or that chess has a *steep learning curve* compared to checkers. On this colloquial interpretation of the phrase, the learning process is often likened to the process of climbing a steep hill, where "steep" refers to the relative difficulty of learning in terms of the steepness of the ascent of the hill.

The German psychologist, Hermann Ebbinghaus, in his seminal (1885) work on memory and learning, used just this notion to describe his data ([1], Fig. 6): "At first the ascent of the curve is very steep, but later on it appears to gradually flatten out." Using himself as a human subject, Ebbinghaus devised and performed a large variety of experiments involving the learning and retention of a sequence of "nonsense syllables," i.e. strings of letters comprised of a vowel sound placed between two consonants. The x-axis, on the graph of the curve Ebbinghaus describes above, represents an increasing number of nonsense syllables in the sequence to be learned, e.g. he performed experiments with sequences of length 7, 12, 16, 24, and 39 nonsense syllables. The y-axis on the graph of this curve represents the average number of repetitions (re-readings) it took, in order for him to correctly repeat (learn) the sequence. For sequences of length 1 to 6 he found that he learned them essentially immediately upon reading them. Sequences of length 7, 12, 16, 24, 39 took an average of 1, 16.6, 30, 44, 55 repetitions, respectively. So, clearly, for this experiment Ebbinghaus showed that the longer the sequence of nonsense syllables to be learned, the more repetitions it takes to correctly repeat or learn the sequence.

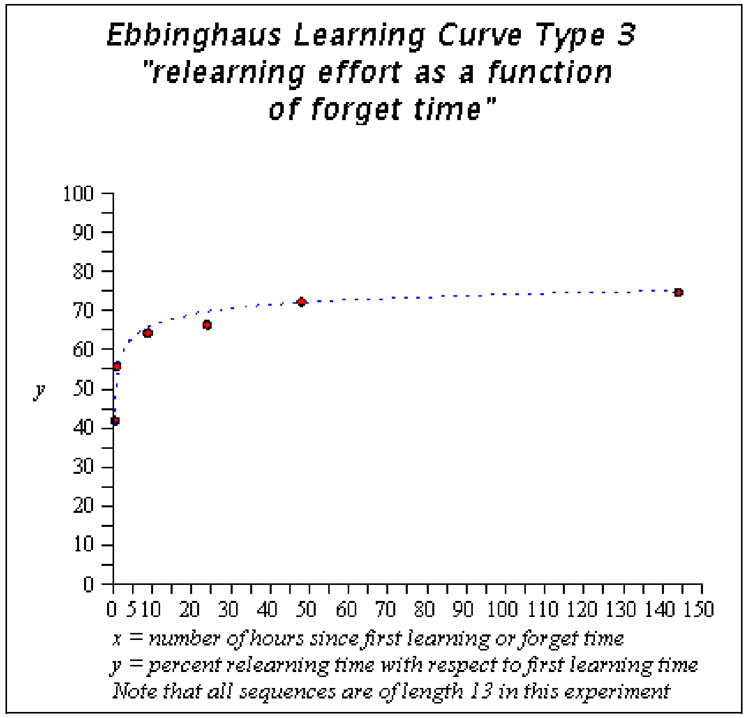


Ebbinghaus is considered to be the orginator of rigorous scientific research in psychology. He is credited with the introduction of well-designed experiments and the use of mathematical analysis, including the creation of a *curve of learning*. In the example discussed above, we see that the curve (right) represents repetitions (y-axis) as a function of length of the sequence (x-axis) or more simply put, "effort as a function of difficulty." Note that for this example, an increase in effort (repetitions) also corresponds implicitly to an increase in total time to successful completion. So, we may also view this curve as representing total time required as a function of difficulty. This last observation seems to capture most of the ordinary usage of “steep learning curve,” since often one is simply asserting that the task at hand has a relatively high difficulty and therefore will require more time.

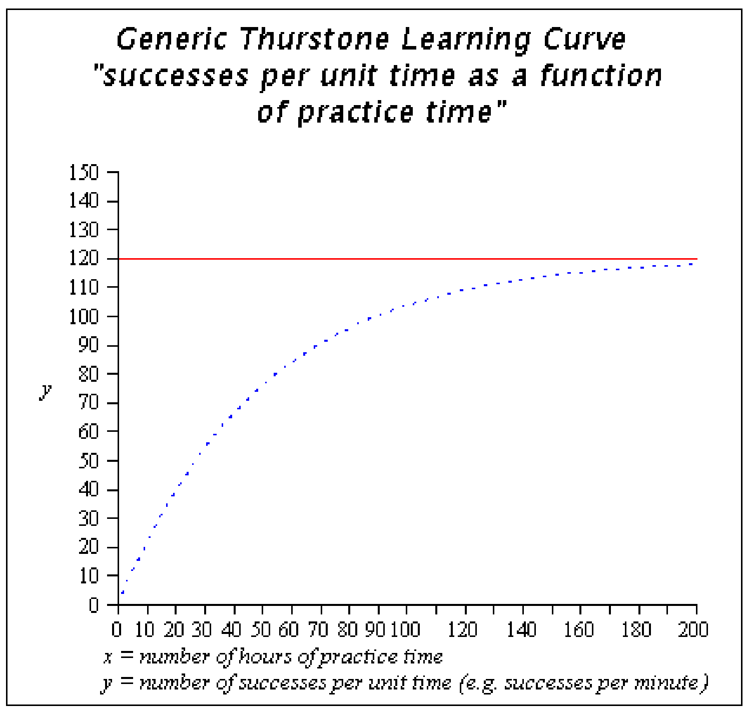


The next relationship that Ebbinghaus considered involved depth of (partial) learning as a function of the number of repetitions. In these experiments he worked only with sequences of nonsense syllables of length sixteen. For these sequences of fixed length he varied the number of repetitions on his initial readings, in order to determine what effect more or less repetitions would have on (learning) relearning them twenty-four hours later. So, the x-axis (right) now contains the number of original repetitions he studied (8, 16, 24, 32, 42, 53, 64), which correspond on the y-axis to the average number of minutes (19.45, 17.97, 16.25, 14.38, 11.67, 9.75, 7.57, respectively) it took him to (learn) relearn twenty-four hours later. In this example, we see that the learning curve (line) represents the time to relearn (y-axis) as a function of the original number of repetitions (x-axis) or more simply put, “relearning effort as a function of original effort.” Suppose that we make the line steeper while constraining it to still pass through the first point on the left. Now, since the line slopes downward from left to right, the steeper the line is, the *easier* the relearning. Hence, for this type of learning curve the interpretation of the phrase “steep learning curve” means essentially the opposite as in our first example. In this case, the steeper the better, i.e. the easier the relearning!

Another interesting experiment that Ebbinghaus performed investigated the relationship between relearning effort and the “forget time,” which he defined as the total elapsed time since the first correct learning. In this experiment all sequences of nonsense syllables were of length thirteen. Ebbinghaus learned these sequences in the usual way, but varied the amount of time he let pass before he attempted to relearn these sequences. Furthermore, he expressed his relearning effort as the percentage of relearning time compared to the first learning time. In this example our x-axis data contains his chosen forget times (1/3, 1, 8.8, 24, 48, 144, 744) in hours. Our y-axis contains his corresponding relearning percentages (41.8, 55.8, 64.2, 66.3, 72.2, 74.6, 78.9, respectively). So, in this case we have percent relearning required expressed as a function of forget time or more simply “relearning effort as a function of forget time.” According to this data, we see that this type of learning curve is very steep initially, but then flattens out quite a bit after about twenty-four hours. The initial steepness indicates how rapidly forgetting occurs within the first twenty-four hours, but then slows down significantly for the remainder of the curve. Of course it makes sense to call this curve a “forgetting curve” and Ebbinghaus should also be credited for his pioneering work on retention as well as learning.



Ebbinghaus studied pure mental processes involving human memory and retention. In contrast, the final example we will discuss addresses simple learning processes that also involve human motor skills such as typing, shooting free throws, or catching and stacking cake boxes, freshly impressed in an assembly line process. In each case, learning can be described in terms of an increase in the number of successes per time as a function of practice time. This type of learning curve was first studied by Louis Leon Thurstone (1917) in his University of Chicago doctoral dissertation (see [2]). The example plot (immediately above) represents a generic Thurstone learning curve, in which successes per unit time are expressed as a function of practice time. This plot could conceivably describe typing skill in words per minute as a function of practice time in hours, with a desired attainment level of one hundred twenty words per minute. Here we have yet another interpretation of “steep learning curve,” which in this context means that



the steeper the curve the faster the learning. As one can readily see, a nice side-effect of this discussion is to provide motivation for the study of curves and their corresponding steepness or slope, which is an intrinsic component of the understanding goals for calculus.

References

[1] Ebbinghaus, H. *Über das Gedächtnis: Untersuchungen zur experimentellen Psychologie.* Leipzig: Duncker & Humbolt, 1885. Online version in English: http://psychclassics.yorku.ca/Ebbinghaus/index.htm

[2] Thurstone, L.L. The Learning Curve Equation, *Psychological Monographs,* Studies from the Psychological Laboratories of the University of Chicago, XXVI (3), pp. 1-51, 1919.