Final Exam Review Part I

Calculus Section E

Fall 2005

Dr. Pais

Find the limit in Problem 1 by applying limit rules step-by-step and labeling each step with the appropriate limit rule.

Margaret

Problem 1. $\lim_{x\to(-2)} \frac{5+2x}{x^3} - 3x =$

1.) $\lim \frac{(5+2x)}{x^3} - \lim 3x$ Difference

2.) $\frac{\lim 5+2x}{\lim x^3} - \lim 3x$ Quotient

3.) $\frac{\lim 5 + \lim 2x}{\lim x^3} - \lim 3x$ Sum

4.) $\frac{\lim 5 + \lim 2x}{(\lim x)^3} - \lim 3x$ Power

5.) $\frac{5+2(-2)}{(-2)^3} - 3(-2)$ Identity

$\frac{5+(-4)}{-8} + 6 \rightarrow \frac{1}{-8} + 6 = 5.87$

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Alana

Problem 2. $\displaystyle\lim_{h\to 0}\frac{\frac{1}{h}-5h}{\frac{2}{h}+5h}=$

$$\lim_{h\to 0}\left[\frac{\frac{1}{\cancel{h}}-5h}{\frac{2}{\cancel{h}}+5h}\right]\frac{\cancel{h}}{\cancel{h}}=\frac{1-5h^2}{2+5h^2}$$

$$\lim_{h\to 0}\text{ IFR }\frac{1-0}{2+0}=\frac{1}{2}$$

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Jeff

Problem 3. $\lim_{x \to 2} \frac{1}{x^2 - 4} =$

$$\frac{\lim 1}{\lim x^2 - \lim 4} = \frac{1}{2(2) - 4}$$

$$\frac{1}{0}$$ undefined, div by 0.

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Kiki

Problem 4. $\lim_{x\to(-4)} \frac{2x^2+5x-12}{x+4} = \frac{\lim_{x\to(-4)} 2x^2+5x-12}{\lim_{x\to(-4)} x+4} =$

$$\frac{\lim_{x\to-4} 2x^2 + \lim_{x\to-4} 5x - \lim_{x\to-4} 12}{\lim_{x\to-4} x + \lim_{x\to-4} 4} = \frac{\lim_{x\to-4} 2x^2 + \lim_{x\to-4} 5x - 12}{\lim_{x\to-4} x + 4} =$$

$$\frac{2(-4)^2+5(-4)-12}{-4+4} = \frac{32-20-12}{0} = \frac{0}{0}$$ doesn't exist

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Meredith

Problem 5. $\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h} = \frac{3(x^2+2xh+h^2)-3x^2}{h}$

$$\frac{\cancel{3x^2}+6xh+3h^2-\cancel{3x^2}}{h}$$

$$\frac{6x\cancel{h}+3\cancel{h}^2}{\cancel{h}}$$

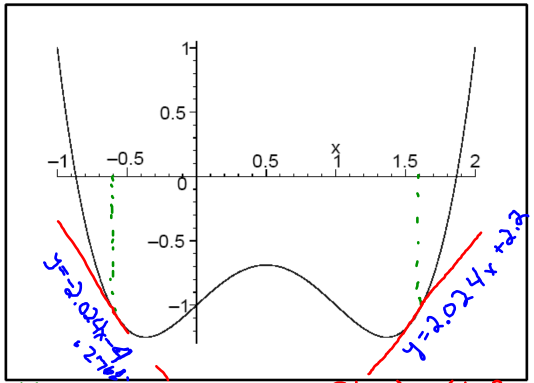
$$\frac{6x+3h}{1} \qquad \lim_{h\to 0} = 6x$$

$$\frac{6x+3(0)}{1}$$

Problem 6. Find the tangent slope function (derivative) for f(x) and use it to find the tangent lines to the curve at the points x = -0.6, 1.6. Also, accurately graph both tangent lines on the plot below.

Garland and Brooke

$f(x) = x^4 - 2x^3 + x - 1$



y-coords

$f(-.6) = x^4 - 2x^3 + x - 1$

$= (-.6)^4 - 2(-.6^3) + (-.6) - 1$

$y = -1.0384$

$(-.6, -1.0384)$

$f(1.6) = (1.6^4) - 2(1.6^3) + 1.6 - 1$

$y = -1.0384$

$(1.6, -1.0384)$

$f'(x) = 4x^3 - 6x^2 + 1$

Slopes → plug points into tsf function (f')

$f'(-.6) = 4x^3 - 6x^2 + 1$

$f'(-.6) = 4(-.6^3) - 6(-.6^2) + 1$

slope $= -2.024$

$f'(1.6) = 4(1.6^3) - 6(-1.6^2) + 1$

slope $= 2.024$

Equations

$y = mx + b$

$-1.0384 = (-2.024)(-.6) + b$

$b = -2.2528$

$y = mx + b$

$(-1.0384) = (2.024)(1.6) + b$

$b = -4.2768$

Final Equations

$y = -2.024x - 2.2528$

$y = 2.024x - 4.2768$

Problem 7. A hot air balloon is ascending at a constant speed of 19 m/s. When it is 33 m above the ground, a care package is released from the balloon.

(a) Specify the position, velocity, and acceleration functions for the motion of the package.

Stephanie

$$s(t) = \frac{1}{2}\, a_0\, t^2 + v_0\, t + s_0$$

Meredith

$$s(t) = \tfrac{1}{2}(9.8)t^2 + 19t + 33$$

$$v = s'(t) = 9.8t + 19$$

$$a = s''(t) = 9.8$$

(b) How long after being released does it take the package to descend to a height of 45 m? What is the velocity of the package when it descends to this height?

Julia

$$45 = -4.9t^2 + 19t + 33$$

$$0 = -4.9t^2 + 19t - 12$$

$$\frac{-19 \pm \sqrt{19^2 - 4(-4.9)(-12)}}{2(-4.9)}$$

$$\frac{-19 \pm \sqrt{125.8}}{-9.8} = 0.794 \text{ or } 3.08$$

$$s'(t) = (-9.8)(3.08) + 19 = -11.18 \text{ m/s}$$

(c) What is the maximum height reached by the package? What is the velocity of the package when it reaches this height?

Mark

$v(t)=0 \quad s'(t)=-9.8t+19$

$(-9.8/2)(1.94)^2+19(1.94)+33=\underline{51.42}\text{ ft.}$

$-19=-9.8t \quad t=1.94\text{ s}$

$-9.8(1.94)+19=-.012\text{ m/s}$

$\approx \underline{\underline{0}}$

(d) What is the velocity of the package just before it hits the ground? What is the maximum speed of the package?

Shaterra

$$0 = -4.9t^2 + 19t + 33$$

$$\frac{-19 \pm \sqrt{(19)^2 - 4(-4.9)(33)}}{2(-4.9)}$$

$$\frac{-19 \pm \sqrt{1007.8}}{-9.8}$$

$$t = 5.1781 \text{ s}$$

$$S'(t) = -9.8(5.1781) + 19$$

$$S'(t) = -31.7459 \text{ m/s}$$

velocity

$$\text{speed} = |\text{velocity}|$$

$$\text{speed} = 31.7459 \text{ m/s}$$

Problem 8.

Emilee

$$f(x) = x^5 - 6x^3 + 7x$$

Find the zeroes of f: -2.1, -1.26, 0, 1.26, 2.1

$u = x^2$

$u^2 = x^4$

$x(x^4 - 6x^2 + 7)$

$u^2 - 6u + 7 = 0$

$$\frac{6 \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$u = 4.415$

$+ 1.585$

$\sqrt{x^2} = \sqrt{4.415} = \pm 2.1$

$\sqrt{x^2} = \sqrt{1.585} = \pm 1.26$

Determine the end behavior of f: down-up

Lindsey

$$f(x) = x^5 - 6x^3 + 7x$$

Find the tangent slope function for f: tsf(x) = $5x^4 - 18x^2 + 7$

Find the zeroes of the tsf function: $-1.78, -.665, .665, 1.78$

$x^2 = u$

$x^4 = u^2$

$5x^4 - 18x^2 + 7$

$5u^2 - 18u + 7 = 0$

$$\frac{18 \pm \sqrt{(-18)^2 - 4(5)(7)}}{2(5)}$$

$u = 3.156$

$.443$

$x = \pm\sqrt{u}$

$x = \pm\sqrt{3.156} = \pm 1.776$

$x = \pm\sqrt{.443} = \pm .665$

Use the information above to sketch the graph of f and to fill in the information below.

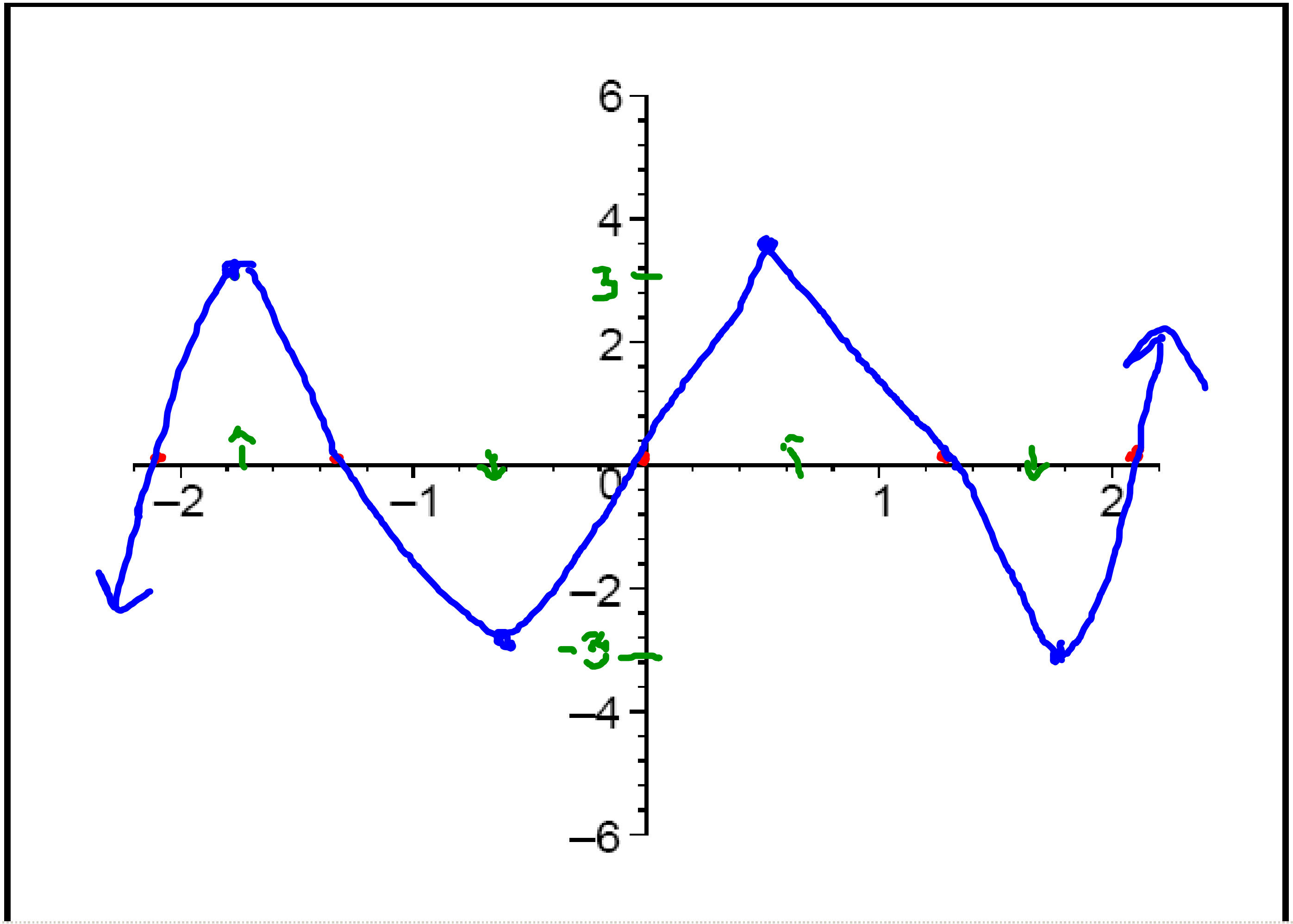
Ali

• $x - intercept(s),\ x =$ -2.1, -1.3, 0, 1.3, 2.1

• $x - coordinate(s)\ of\ bump(s)\ on\ f,\ x =$ -1.777, -.666, .666, 1.777

• $bump1\ on\ f =$ (-1.777, 3.51), $bump2\ on\ f =$ (-.666, -3.02), $bump3\ on\ f =$ (.666, 3.02)

Bump4 on F = (1.777, -3.51)



Problem 9. Compute the limit below for the given function f(x), carefully showing all your work.

Sam

$$f(x) = -2\,x^2 + 5\,x - 1,\ f\,'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= -2(x+h)^2 + 5(x+h) - 1 - (-2x^2 + 5x - 1)$$

$$= -2(x^2 + 2xh + h^2) + 5x + 5h - 1 - (\ \text{''}\ )$$

$$= \cancel{-2x^2} - 4xh - 2h^2 + \cancel{5x} + 5h - \cancel{1} + \cancel{2x^2} - \cancel{5x} + \cancel{1}$$

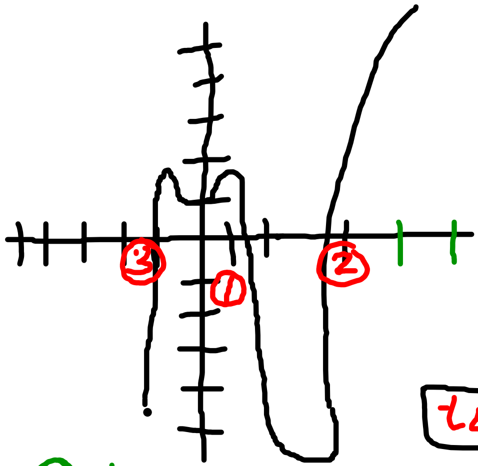
$$= \frac{-4xh - 2h^2 + 5h}{h}$$

$$= \frac{\cancel{h}(-4x - 2h + 5)}{\cancel{h}} = \lim_{h\to 0} -4x - 2h + 5 = \boxed{-4x + 5}$$

Problem 10. Use the Newton-Raphson method to find the real zeroes of f(x), carefully showing all your work.

Margaret and Alana

$$f(x) = x^5 - 3x^4 + 2x^2 + 1$$



$f'(x) = 5x^4 - 12x^3 + 4x$

① $t_0 = 1$

$t_1 = 1.3333 \rightarrow f(t_1) = -.71193416$

$t_2 = 1.235923 \rightarrow f(t_2) = -.0610654$

$t_3 = 1.225820776 \rightarrow f(t_3) = -.000675708$

$\boxed{t_4 = 1.225706457} \rightarrow f(t_4) = -.000000806194$

② $t_0 = 2.5$

$t_1 = 2.83859665 \rightarrow f(t_1) = 6.636072$

$t_2 = 2.7307158 \rightarrow f(t_2) = .94053981$

$t_3 = 2.70962899 \rightarrow f(t_3) = .031359197$

$* \boxed{t_4 = 2.70887585} \rightarrow f(t_4) = .00003899938$

③ $t_0 = -1$

$t_1 = -.9230769231 \quad f(t_1) = -.144109078$

$t_2 = -.9077070723 \quad f(t_2) = -.0049379951$

$t_3 = -.9071419654 \quad f(t_3) = -.0000064823561$

Problem 11. Compute the derivative using the appropriate D-Rules.

Jeff

$D((x^2+2x-2)(x^3-1)) = (2x+2)(x^3-1)+(3x^2)(x^2+2x-2)$

$2x^4-2x+2x^3-2+3x^4+6x^3-6x^2$

$5x^4+8x^3-6x^2-2x-2$

Kiki

$$D\left(\frac{(x^2+2x-2)^2}{x^3-1}\right) = \frac{D[(x^2+2x-2)^2](x^3-1) - D[x^3-1](x^2+2x-2)}{(x^3-1)^2} =$$

$$\frac{2(x^2+2x-2)D[x^2+2x-2](x^3-1) - 3x^2(x^2+2x-2)^2}{(x^3-1)^2} =$$

$$\frac{(x^2+2x-2)\left(2(2x+2)(x^3-1) - 3x^2(x^2+2x-2)\right)}{(x^3-1)^2} =$$

$$\frac{(x^2+2x-2)((4x+4)(x^3-1) - 3x^4 - 6x^3 + 6x^2)}{(x^3-1)^2} =$$

$$\frac{(x^2+2x-2)((4x^4-4x+4x^3-4) - 3x^4 - 6x^3 + 6x^2)}{(x^3-1)^2} =$$

$$\frac{(x^2+2x-2)(x^4-2x^3+6x^2-4x-4)}{(x^3-1)^2}$$

Meredith

$$D\left(\left(\frac{\cos(x)}{e^x}\right)^5\right) = 5\left(\frac{\cos(x)}{e^x}\right)^4 \cdot D\left[\frac{\cos(x)}{e^x}\right]$$

$$= \frac{\left(D[\cos(x)] \cdot e^x\right) - \left(D[e^x] \cdot \cos(x)\right)}{(e^x)^2}$$

$$= \frac{\left(-\sin(x) \cdot e^x\right) - \left(e^x \cdot \cos(x)\right)}{(e^x)^2}$$

Garland

$D\left(e^{x}+x\ln(x)\right)=$

$D(e^{x})+D(x\ln x)$

$e^{x}+D(x)(\ln x)+D(\ln x)(x)$

$e^{x}+1\cdot\ln x+\frac{1}{x}\cdot x$

$e^{x}+\ln x+1$

Brooke

$$D\left(\ln\left(\frac{x}{\sin(x)+1}\right)\right) = \frac{D\left(\frac{x}{\sin(x)+1}\right)}{\frac{x}{\sin(x)+1}} = \frac{\frac{D(x)(\sin x+1)-D(\sin x+1)(x)}{(\sin x+1)^2}}{\frac{x}{\sin(x)+1}}$$

$$= \frac{1(\sin x+1)-[D(\sin x)+D(1)]x}{} = \frac{\frac{\sin x+1-[(\cos x-0)x]}{(\sin x+1)^2}}{\frac{x}{\sin(x+1)}}$$

$$\frac{(\sin x+1)}{x}\cdot\frac{(\sin x+1)-[x\cos x]}{(\sin x+1)^2}$$

Stephanie Garland

$$D\left(e^{\cos(x^{-2})} + (x^4 - x^2 + 1)^{-3}\right) =$$

$$D\left(e^{\cos(x^{-2})}\right) + D\left[(x^4 - x^2 + 1)^{-3}\right]$$

$$e^{\cos(x^{-2})} \cdot D(\cos(x^{-2})) + -3(x^4 - x^2 + 1)^{-4} \cdot D(x^4 - x^2 + 1)$$

$$'' \quad -\sin x^{-2} \cdot D(x^{-2}) + \quad '' \quad \cdot (4x^3 - 2x)$$

$$e^{\cos(x^{-2})} \cdot (-\sin(x^{-2})) \cdot (-2x^{-3}) - 3(x^4 - x^2 + 1)^{-4} \cdot (4x^3 - 2x)$$

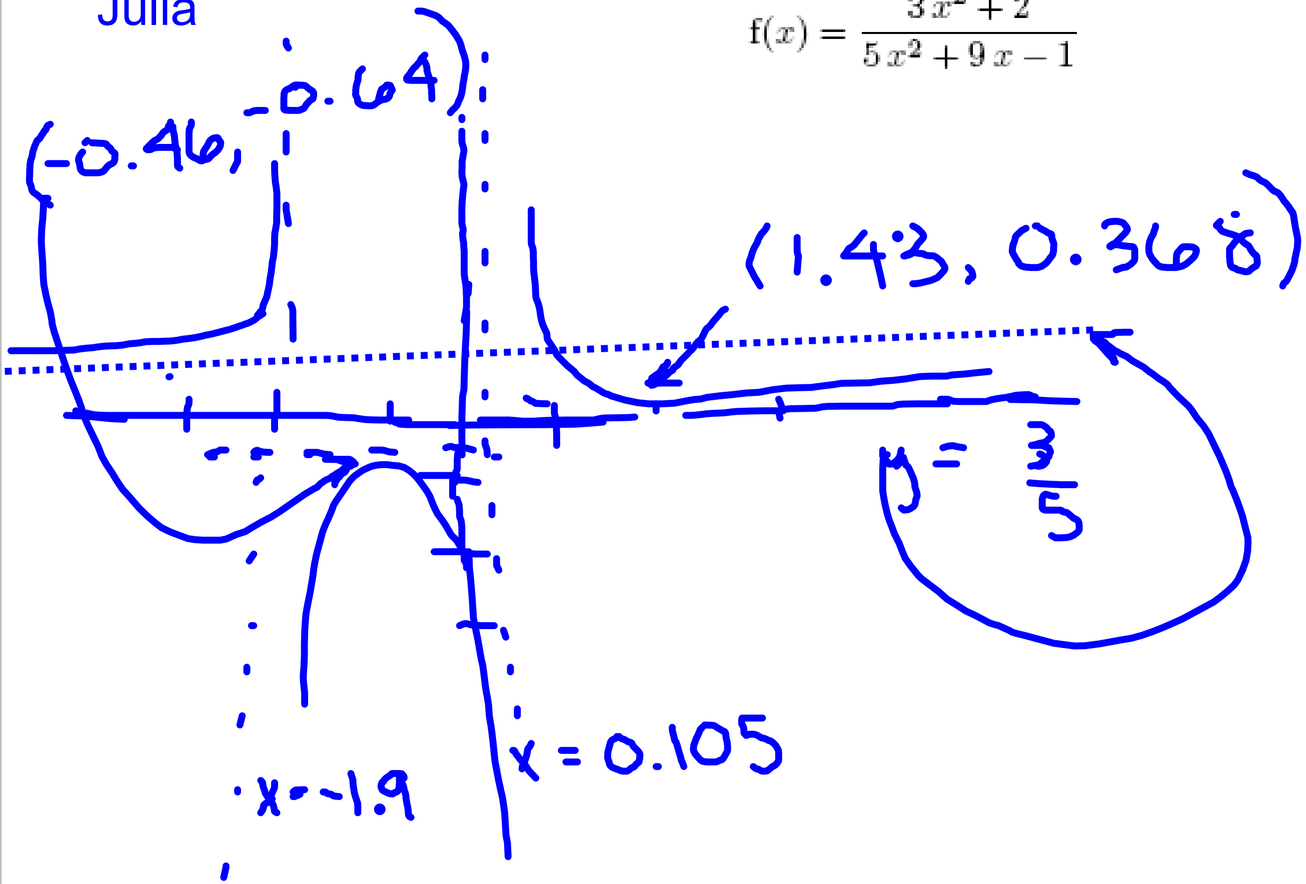
Problem 12.

1. Sketch an accurate graph of the function f(x),

6. **Label** the graph in 1. with the information in 2., 4. and 5. (After they are completed below.)

Julia

$$f(x) = \frac{3\,x^2+2}{5\,x^2+9\,x-1}$$



2. Find all x-intercepts,

Mark

$$f(x) = \frac{3\,x^2 + 2}{5\,x^2 + 9\,x - 1}$$

$$3x^2 + 2 = 0$$

$$3x^2 = -2$$

$$x^2 = -2/3$$

$$x = \sqrt{-2/3} \rightarrow \text{imaginary \#.}$$

no x-int.

3. Compute the derivative,

4. Find the xy-coordinates of all bump points,

Shaterra

$$f(x) = \frac{3x^2+2}{5x^2+9x-1}$$

3. $D\left[\frac{3x^2+2}{5x^2+9x-1}\right] =$

$$\frac{\left(D[3x^2+2]\cdot(5x^2+9x-1)\right) - \left(D[5x^2+9x-1]\cdot(3x^2+2)\right)}{(5x^2+9x-1)^2}$$

$$((6x)(5x^2+9x-1)) - ((10x+9)(3x^2+2))$$

$$(30x^3+54x^2-6x) - (30x^3+20x+27x^2+18)$$

$$\frac{30x^3+54x^2-6x-30x^3-20x-27x^2-18}{(5x^2+9x-1)^2}$$

$$\frac{27x^2-26x-18}{(5x^2+9x-1)^2} = f'(x)$$

4. $0 = 27x^2-26x-18$

$$\frac{26 \pm \sqrt{(-26)^2-4(27)(-18)}}{2(27)}$$

$$\frac{26 \pm \sqrt{2620}}{54}$$

$x = 1.4294$ & -0.4664

$$\frac{3(1.4294)^2+2}{5(1.4294)^2+9(1.4294)-1} = .36816$$

Bump 1 = (1.429, .3682)

$$\frac{3(-.4664)^2+2}{5(-.4664)^2+9(-.4664)-1} = -.6454$$

Bump 2 = (-.4664, -.6454)

5. Find all vertical and horizontal asymptotes,

Emilee

$$f(x) = \frac{3\,x^2 + 2}{5\,x^2 + 9\,x - 1}$$

vertical -

$$5x^2+9x-1=0$$

$$\frac{-9 \pm \sqrt{(9)^2-4(5)(-1)}}{2(5)}$$

$$-\frac{9}{10} \pm \frac{\sqrt{101}}{10} = .105 \;\&\; -1.91$$

horizontal -

$$\lim_{x \to \infty} \frac{3x^2+2}{5x^2+9x-1} \quad \frac{\frac{3x^2+2}{x^2}}{\frac{5x^2+9x-1}{x^2}} \quad \frac{3+2/x^2}{5+\frac{9x}{x^2}-1/x^2}$$

$$\boxed{\frac{3}{5}=y}$$