Final Exam Review Part I

Calculus Section F

Fall 2005

Dr. Pais

Find the limit in Problem 1 by applying limit rules step-by-step and labeling each step with the appropriate limit rule.

Carolyn

Problem 1. $\lim_{x\to(-2)} \frac{5+2x}{x^3} - 3x =$

$$\frac{\lim 5 + \lim 2x}{\lim x^3} - \lim 3x$$

$$\frac{5+(-4)}{-8} - (-6)$$

$$-\frac{1}{8} + 6 = \frac{47}{8} = 5.857$$

$\lim \left(\frac{5+2x}{x^3}\right) - \lim 3x$ DR

$\lim \frac{(5+2x)}{\lim x^3} - \lim 3x$ QR

$\frac{\lim 5 + \lim 2x}{(\lim x)^3} - \lim 3x$ SR, PR

$\frac{5+2(-2)}{(-2)^3}$ - " CFR IFR

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Sarah

Problem 2. $\lim_{h \to 0} \dfrac{\frac{1}{h} - 5\,h}{\frac{2}{h} + 5\,h} =$

$$= \frac{\frac{1}{h} - 5h}{\frac{2}{h} + 5h} \cdot \frac{h}{h}$$

$$= \frac{1-5h^2}{2+5h^2} \Rightarrow \lim_{h \to 0} \frac{1-\underbrace{5h^2}_{\to 0}}{2+\underbrace{5h^2}_{\to 0}} = \frac{1}{2}$$

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Brittany

Problem 3. $\lim_{x \to 2} \frac{1}{x^2 - 4} =$

$$\frac{\lim 1}{\lim x^2 - \lim 4} = \frac{1}{(2)^2 - 4} = \frac{1}{0} \text{ undefined}$$

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Emily

Problem 4. $\lim_{x \to (-4)} \frac{2x^2 + 5x - 12}{x + 4} =$

$$\frac{(2x-3)\cancel{(x+4)}}{\cancel{(x+4)}} = 2x - 3$$

$$\lim_{x \to (-4)} 2x - \lim_{x \to (-4)} 3$$

$$-8 - 3 = -11$$

In Problems 2-5, if possible, find the limit carefully showing all your work. Otherwise, explain clearly why the limit does not exist.

Scott

Problem 5. $\lim_{h\to 0} \dfrac{3\,(x\ +h)^2 - 3\,x^2}{h} =$

$$\frac{3(x^2+xh+xh+h^2)-3x^2}{h}$$

$$3x^2+6xh+3h^2-3x^2$$

$$\lim_{h\to 0}\frac{6xh+3h^2}{h}$$

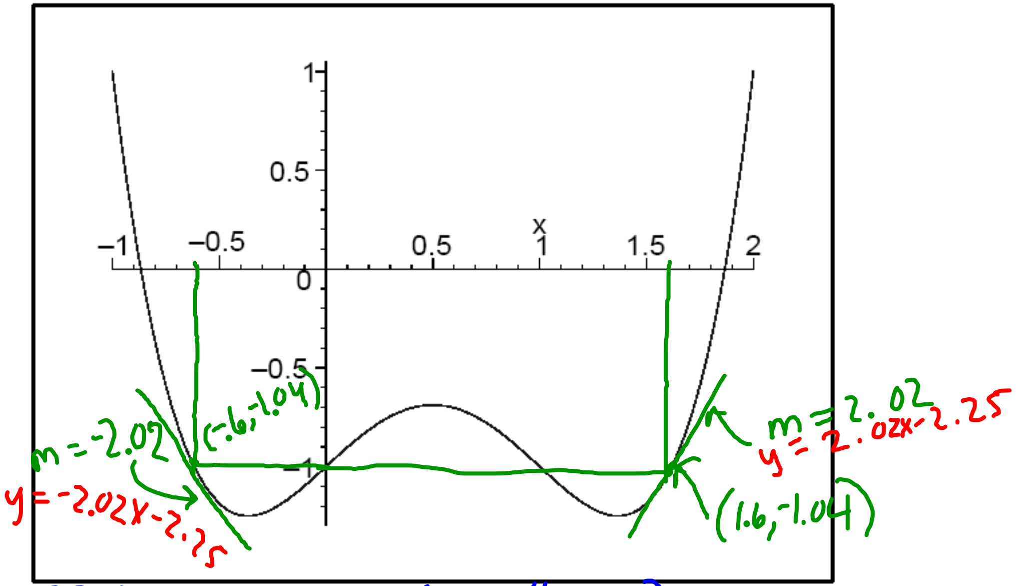
$$\lim_{h\to 0} 6x+3h = \longrightarrow \lim_{h\to 0} 6x + \lim_{h\to 0} 3h = 6x$$

goes to zero

Problem 6. Find the tangent slope function (derivative) for f(x) and use it to find the tangent lines to the curve at the points x = -0.6, 1.6. Also, accurately graph both tangent lines on the plot below.

Madeline and Samah

$f(x) = x^4 - 2x^3 + x - 1$



Mdawg

$f(x) = x^4 - 2x^3 + x - 1$

$tsf = f'(x) = 4x^3 - 6x^2 + 1$

$f'(x) = 4(-.6)^3 - 6(-.6)^2 + 1$

slope $= -2.024$

$f(x) = (-.6)^4 - 2(-.6)^3 + (-.6) - 1$

$= -1.04$

$f'(x) = 4(1.6)^3 - 6(1.6)^2 + 1$

slope $= 2.024$

$f(x) = (1.6)^4 - 2(1.6)^3 + 1.6 - 1$

$= -1.04$

$-1.04 = 2.02(1.6) + b$

$b = -2.25$

$y = 2.02x - 2.25$

$-1.04 = -2.02(-.6) + b$

$b = -2.25$

$y = -2.02x - 2.25$

Problem 7. A hot air balloon is ascending at a constant speed of 19 m/s. When it is 33 m above the ground, a care package is released from the balloon.

(a) Specify the position, velocity, and acceleration functions for the motion of the package.

Burke

$$s(t) = \frac{1}{2} a_0 t^2 + v_0 t + s_0$$

$$s = -4.9t^2 + 19t + 33$$

$$v = -9.8(t) + 19$$

$$a = -9.8 \, m/s^2$$

(b) How long after being released does it take the package to descend to a height of 45 m? What is the velocity of the package when it descends to this height?

Chloe

$$45 = -4.9t^2 + 19t + 33$$

$$-4.9t^2 + 19t - 12 = 0$$

$$x = \frac{-19 \pm \sqrt{(19)^2 - 4(-4.9)(-12)}}{2(-4.9)}$$

$$= \frac{-19 + 11.22}{-9.8}$$

$$= .79$$

$$= \frac{-19 - 11.22}{-9.8}$$

$$= 3.08$$

descending

$$-9.8(3.08) + 19$$

$$= -11.184$$

(c) What is the maximum height reached by the package? What is the velocity of the package when it reaches this height?

Carolyn

$V(t)=0$

$0=-9.8t+19$

$t=1.93\ s$

$S(1.93)=-4.9(1.93)^2+19(1.93)+33$

$51.42\ m$

(d) What is the velocity of the package just before it hits the ground? What is the maximum speed of the package?

Sarah

$S(t) = -4.905t^2 + 19t + 33$

$S'(t) = -9.81t + 19$

$S''(t) = -9.81$

$0 = -4.905t^2 + 19t + 33$

$t = 5.174s$

$(-9.81)(5.14) + 19 = -31.75694$ m/s

MAX Speed $= |-31.75694| = 31.7569$ m/s

Problem 8.

Brittany

$$f(x) = x^5 - 6x^3 + 7x$$

Find the zeroes of f: $x = 0, -2.101, 2.101, -1.259, 1.259$

$f(x) = 0$

$x^5 - 6x^3 + 7x$

$x(x^4 - 6x^2 + 7)$

$u = x^2$ $u^2 - 6u + 7 = 0$

$= \dfrac{6 \pm \sqrt{6^2 - 4 \cdot 7 \cdot 1}}{2}$

$= \dfrac{6 \pm \sqrt{8}}{2}$

$= \dfrac{6 + 2.828}{2}$

$u = 4.414$

$x = \pm 2.101$

$= \dfrac{6 - 2.828}{2}$

$u = 1.586$

$x = \pm 1.293$

Determine the end behavior of f: Down-Up

Emily $\quad f(x) = x^5 - 6x^3 + 7x$

Find the tangent slope function for f: tsf(x) = $5x^4 - 18x^2 + 7$

Find the zeroes of the tsf function: $-1.77, -.666, .666, 1.77$

$5x^4 - 18x^2 + 7 = 0$

$x^2 = u$

$5u^2 - 18u + 7 = 0$

$u = \dfrac{18 \pm \sqrt{(-18^2) - (4 \cdot 5 \cdot 7)}}{2(5)}$

$u = 3.156$

$x = \sqrt{u}$

$x = \pm 1.77$

$u = .444$

$x = \sqrt{u}$

$x = \pm .666$

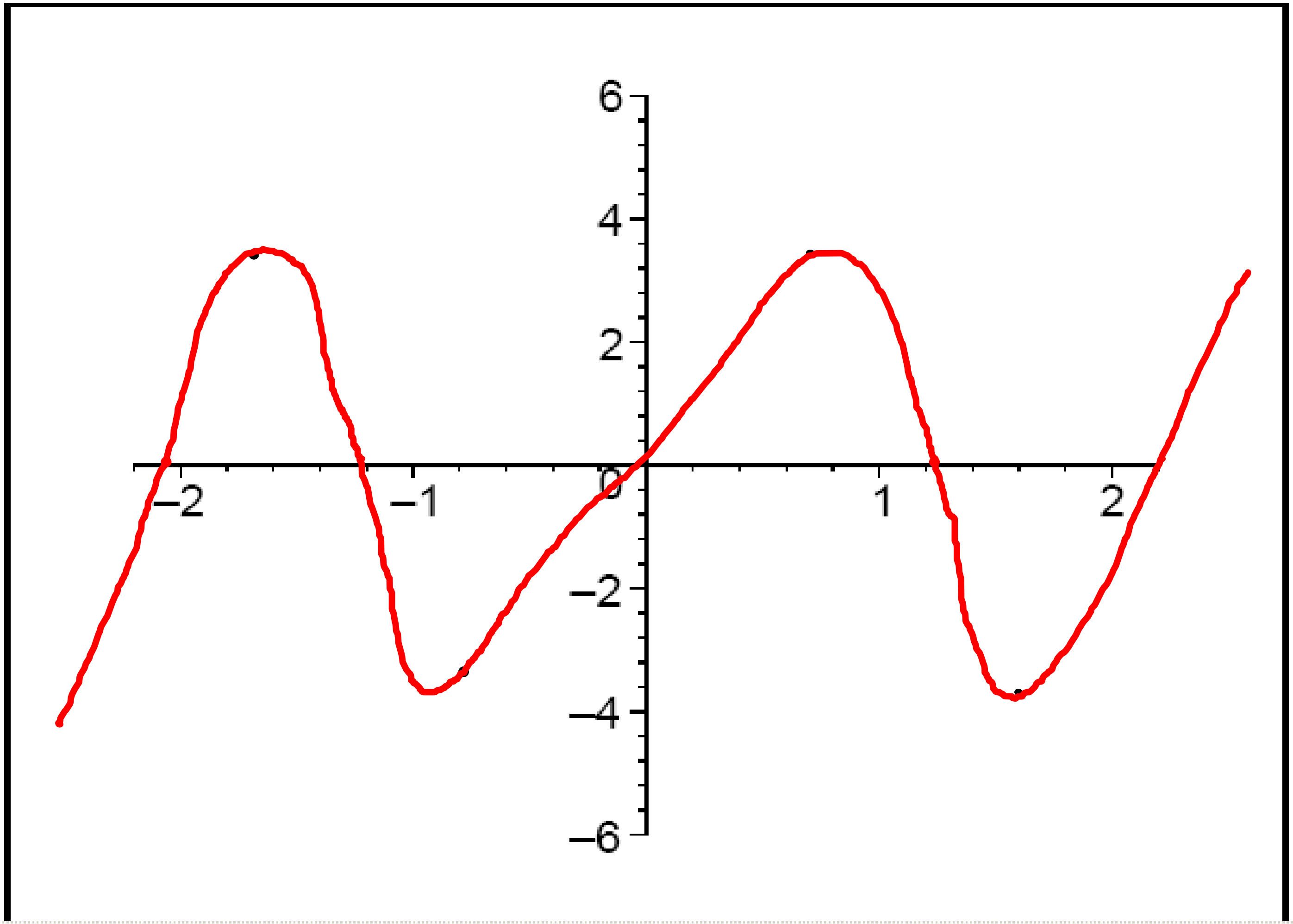
Use the information above to sketch the graph of f and to fill in the information below.

Scott

$x - intercept(s),\quad x =$ 0, -2.01, 2.01, 1.259, -1.259

$x - coordinate(s)\ of\ bump(s)\ on\ f,\ x =$ (1.77, -3.5)

$bump1\ on\ f =$ (.666, 3.02), $bump2\ on\ f =$ (-.666, -3.02), $bump3\ on\ f =$ (-1.77, 3.5)



Problem 9. Compute the limit below for the given function f(x), carefully showing all your work.

Madeline

$$f(x) = -2\,x^2 + 5\,x - 1,\ f\,'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

no QR can't be real

$$= -2(x+h)^2 + 5(x+h) - 1 - (-2x^2 + 5x - 1)$$

$$= -2(x^2 + 2xh + h^2) + \cancel{5x} + 5h - 1 + 2x^2 - \cancel{5x} + 1$$

$$= -2x^2 - 4xh - 2h^2 + 5h - 1 + 2x^2 + 1$$

$$= -4xh - 2h^2 + 5h$$

$$= \frac{\cancel{h}(-4x - 2h + 5)}{\cancel{h}} \Rightarrow \lim_{h \to 0} -4x - 2\cancel{h} + 5$$

0

$$= -4x + 5$$

Problem 10. Use the Newton-Raphson method to find the real zeroes of f(x), carefully showing all your work.

Samah

$$f(x) = x^5 - 3x^4 + 2x^2 + 1$$

$D[f(x)] = 5x^4 - 12x^3 + 4x$

begin guess → $t_0 = -.9$; $f(t_0) = -.907262$

$t_1 = -.907262$; $f'(t_1) = -.907141$

$\downarrow$

$t_2 = -.907141$ $f(-.907141) = 0$

begin guess → $t_0 = 1.2$; $f(t_0) = 1.22649$

$t_1 = 1.22649$; $f(t_1) = 1.22571$

$\downarrow$

$f(1.22571) = -.000021 \approx 0$

begin guess → $t_0 = 2.7$

$\downarrow$ 24 steps

$t_{24} = 2.70885$

$f(2.70885) \approx 0$

Problem 11. Compute the derivative using the appropriate D-Rules.

Burke

$D((x^2+2x-2)(x^3-1)) =$

$= D(x^2+2x-2)(x^3-1) + D(x^3-1)(x^2+2x-2)$

$= (2x+2)(x^3-1) + (3x^2)(x^2+2x-2)$

$= 2x^4 - 2x + 2x^3 - 2 + 3x^4 + 6x^3 - 6x^2$

$= 5x^4 + 8x^3 - 6x^2 - 2x - 2$

Chloe

$$D\left(\frac{(x^2+2x-2)^2}{x^3-1}\right) = \frac{D[(x^2+2x-2)^2](x^3-1) - D[x^3-1](x^2+2x-2)^2}{(x^3-1)^2}$$

$$= \frac{2(x^2+2x-2)\cdot D[x^2+2x-2](x^3-1) - (3x^2)(x^2+2x-2)^2}{(x^3-1)^2}$$

$$= \frac{2(x^2+2x-2)(2x+2)(x^3-1) - \text{''} \quad \text{''}}{\text{''}}$$

$$= \frac{2(x^2+2x-2)(3x^4+2x^3-2+2x^3) - (3x^2)(x^2+2x-2)}{}$$

$$= \frac{\text{''} - (3x^4+6x^3-6x^2)}{}$$

$$= \frac{2(x^2+2x-2)(-4x^3+2x^3-2) - (-6x^2)}{\text{''}}$$

Carolyn

$$D\left(\left(\frac{\cos(x)}{e^x}\right)^5\right) = 5\left(\frac{\cos(x)}{e^x}\right)^4 \cdot D\left[\frac{\cos(x)}{e^x}\right]$$

$$= \frac{D[\cos(x)](e^x) - D[e^x][\cos(x)]}{(e^x)^2}$$

$$= \frac{-\sin(x)e^x - \cos(x)e^x}{(e^x)^2}$$

Sarah

$D\left(e^x + x\ln(x)\right) =$

$D(e^x + x\ln(x)) = D(e^x) + [D(x)(\ln(x)) + D(\ln(x))(x)]$

$= e^x + [(\ln(x)) + (\frac{1}{x})(x)]$

$= e^x + \ln(x) + 1$

Brittany

$$D\left(\ln\left(\frac{x}{\sin(x)+1}\right)\right) = \frac{1}{\frac{x}{\sin(x)+1}} = \frac{\sin x+1}{x}$$

$$D\left[\frac{x}{\sin(x)+1}\right] = \text{"}\left(\frac{D(x)(\sin(x)+1) - D(\sin x+1)(x)}{(\sin x+1)^2}\right)$$

$$= \text{"}\left(\frac{1(\sin x+1) - \cos x(x)}{(\sin x+1)^2}\right)$$

$$= \left(\frac{\sin x+1}{x}\right)\left(\frac{\sin x+1-\cos(x)(x)}{(\sin x+1)^2}\right)$$

Emily

$$D\left(e^{\cos(x^{-2})} + (x^4 - x^2 + 1)^{-3}\right) = D[e^{\cos x^{-2}}] + D[(x^4 - x^2 + 1)^{-3}]$$

$$e^{\cos x^{-2}} \cdot D[\cos x^{-2}] + [(-3(x^4 - x^2 + 1)^{-4}) \cdot D[x^4 - x^2 + 1]]$$

$$e^{\cos x^{-2}} \cdot (-\sin x^{-2}) \cdot D[x^{-2}] + [(-3(x^4 - x^2 + 1)^{-4}) \cdot (4x^3 - 2x)]$$

$$e^{\cos x^{-2}} \cdot (-2x^{-3})(-\sin x^{-2}) + (-3(x^4 - x^2 + 1)^{-4} \cdot (4x^3 - 2x))$$

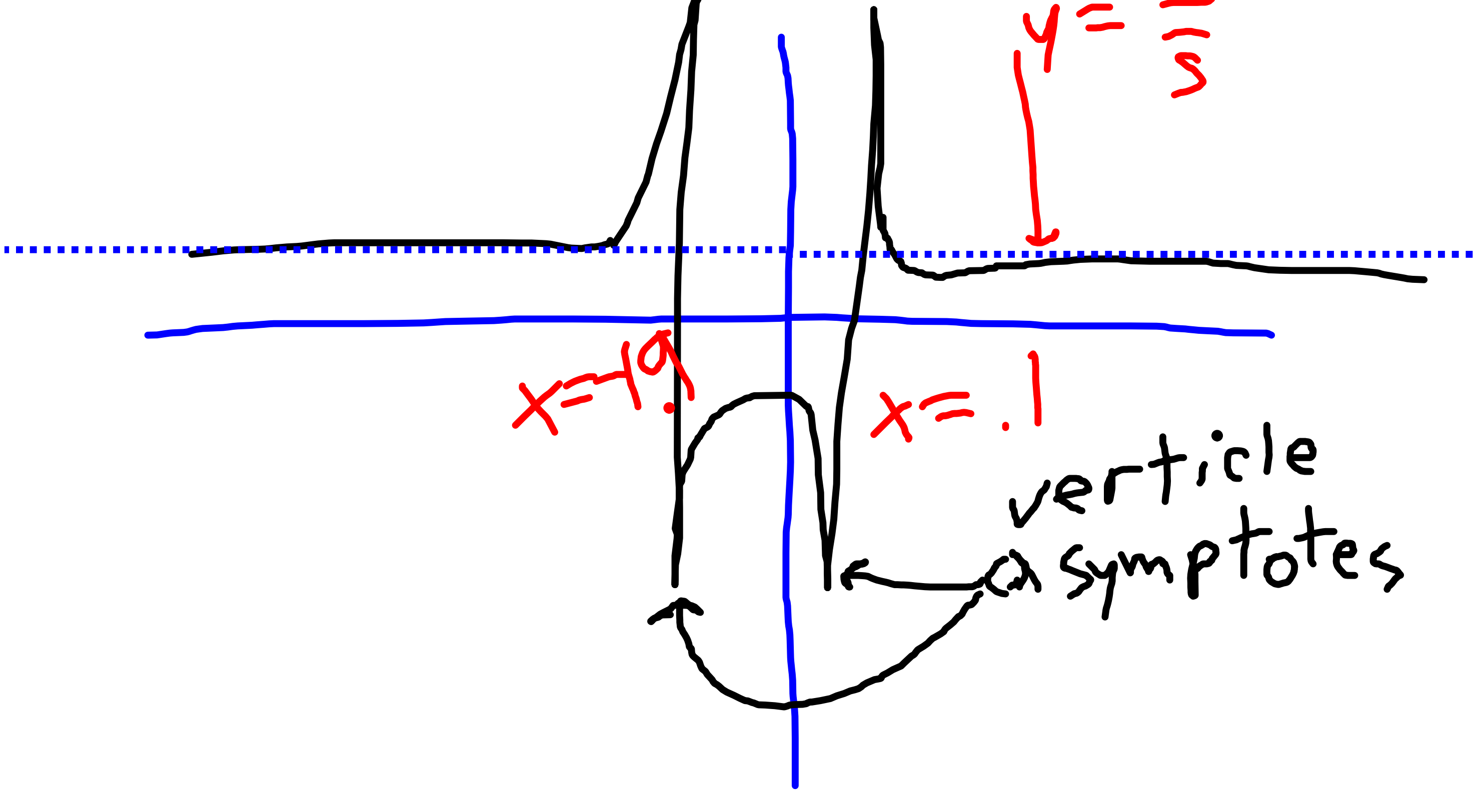
Problem 12.

1. Sketch an accurate graph of the function f(x),

6. **Label** the graph in 1. with the information in 2., 4. and 5. (After they are completed below.)

Scott

$$f(x) = \frac{3\,x^2+2}{5\,x^2+9\,x-1}$$



2. Find all x-intercepts,

Madeline

$$f(x) = \frac{3\,x^2 + 2}{5\,x^2 + 9\,x - 1}$$

$3x^2+2=0$

$\sqrt{x^2} = \sqrt{-\frac{2}{3}}$

imaginary,
no real roots

3. Compute the derivative,

4. Find the xy-coordinates of all bump points,

Samah

$$f(x) = \frac{3x^2+2}{5x^2+9x-1}$$

$$D\left[\frac{3x^2+2}{5x^2+9x-1}\right]$$

$$\to \frac{D[3x^2+2](5x^2+9x-1)-D[5x^2+9x-1](3x^2+2)}{(5x^2+9x-1)^2}$$

$$= \frac{(6x)(5x^2+9x-1)-(10x+9)(3x^2+2)}{(5x^2+9x-1)^2}$$

$$0=(6x)(5x^2+9x-1)-(10x+9)(10x^2+2)$$

$$\to x = -2$$

$$(x, f(x)) \to (2, f(-2))$$

$$=(-2, 14)$$

5. Find all vertical and horizontal asymptotes,

Burke and Chloe

$$f(x) = \frac{3x^2+2}{5x^2+9x-1}$$

Horizontal

$$\lim_{x\to\infty} \frac{3x^2+2}{5x^2+9x-1} \qquad y = \frac{3}{5}$$

$$\frac{3 + \frac{2}{x^2} \to 0}{5 + \frac{9}{x} + \frac{-1}{x^2} \to 0}$$

Divide by highest power.

Vertical

$$5x^2+9x-1=0$$

$$x = \frac{-9 \pm \sqrt{9^2-4(4.5\cdot -1)}}{2\cdot 5}$$

$$x = \frac{-9 \pm \sqrt{101}}{10}$$

$$= .1049 \quad , \quad -1.9049$$