Final Exam Review Part III

Calculus Section F

Fall 2005

Dr. Pais

Green Problem 1. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{(7x^2+x)}\right) =$$

$$e^{(7x^2+x)} D(7x^2+x)$$

$$= e^{(7x^2+x)}(14x+1)$$

Green Problem 2. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{\left(x^{\left(\frac{-5}{3}\right)}\right)}\right) =$$

$$= e^{\left(x^{-\frac{5}{3}}\right)} D\left(x^{-\frac{5}{3}}\right)$$

$$= e^{x^{-\frac{5}{3}}}\left(-\frac{5}{3}x^{-\frac{8}{3}}\right)$$

Green Problem 3. Compute the derivative using the appropriate D-Rules.

$$D(e^{\cos(x)}) =$$

$$e^{\cos(x)}\left(D\ \cos(x)\right)$$

$$= e^{\cos(x)}(-\sin(x))$$

green

Green Problem 4. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{(\sin(x^2-5x-6))}\right) =$$

$$e^{\sin(x^2-5x-6)} \cdot D[\sin(x^2-5x-6)]$$

$$e^{\sin(x^2-5x-6)} \cdot \cos(x^2-5x-6) \cdot D[x^2-5x-6]$$

$$e^{\sin(x^2-5x-6)} \cdot \cos(x^2-5x-6)(2x-5)$$

Green Problem 5. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{\left(e^{x}\right)}\right) =$$

$$e^{e^{x}} \cdot D\left[e^{x}\right]$$

$$e^{e^{x}} \cdot e^{x}$$

Green Problem 6. Compute the derivative using the appropriate D-Rules.

$D(x^{\pi}) =$

$\pi x^{(\pi - 1)}$

Green Problem 7. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{\left(\frac{\sin(x)}{\ln(x)}\right)}\right) =$$

$$= e^{\left(\frac{\sin x}{\ln x}\right)} D\left(\frac{\sin x}{\ln x}\right)$$

$$= \text{''} \cdot \left(\frac{D(\sin x)(\ln x) - D(\ln x)(\sin x)}{(\ln x)^2}\right)$$

$$= \text{''} \cdot \left(\frac{\cos x(\ln x) - \frac{\sin x}{x}}{(\ln x)^2}\right)$$

Green Problem 8. Compute the derivative using the appropriate D-Rules.

$$D(\ln(e^x + 1)) =$$

$$\frac{1}{e^x + 1} D[e^x + 1] =$$

$$\frac{e^x}{e^x + 1}$$

Green Problem 9. Compute the derivative using the appropriate D-Rules.

$D(\ln(\ln(x))) =$

$$\frac{1}{\ln x} D[\ln x] =$$

$$\frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Green Problem 10. Compute the derivative using the appropriate D-Rules.

$D(\ln(\sqrt{x})) =$

$$D\left[\ln\left(x^{\frac{1}{2}}\right)\right] =$$

$$\frac{1}{x^{\frac{1}{2}}} D\left[x^{\frac{1}{2}}\right] =$$

$$\frac{1}{x^{\frac{1}{2}}} \cdot \frac{1}{2} x^{-\frac{1}{2}} =$$

$$\frac{1}{2x^{\frac{1}{2}} \cdot x^{1/2}} = \frac{1}{2x}$$

Red Problem 1. Compute the derivative using the appropriate D-Rules.

$D(e^{(5x)}) =$

$(e^{5x})(5) = 5e^{(5x)}$

Red Problem 2. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{\left(x^{\left(\frac{1}{3}\right)}\right)}\right) =$$

$$= D\left[e^{\sqrt[3]{x}}\right]$$

$$= e^{\sqrt[3]{x}} \cdot D\left(\sqrt[3]{x}\right)$$

$$= e^{x^{\frac{1}{3}}} \cdot x^{-\frac{2}{3}}$$

$$e^{x^{1/3}} \cdot \frac{1}{3}x^{-2/3}$$

Red Problem 3. Compute the derivative using the appropriate D-Rules.

$$D(e^{\sin(x)}) =$$

$$e^{\sin x} \cos x$$

Red Problem 4. Compute the derivative using the appropriate D-Rules.

$$D(e^{(x\sin(x))}) =$$

$$[e^{x\sin x}](D[x](\sin x) + D[\sin x](x))$$

$$(e^{x\sin x})(\sin x + \cos(x)(x))$$

Red Problem 5. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{(x^3\cos(x^2))}\right) =$$

$$e^{(x^3\cos(x^2))} \cdot D[x^3 \cdot (\cos x^2)]$$

$$= \cdot \left(3x^2(\cos x^2) + (-\sin x^2)(x^3)(2x)\right)$$

Red Problem 6. Compute the derivative using the appropriate D-Rules.

$D(x^e) =$

$e x^{(e-1)}$

Red Problem 7. Compute the derivative using the appropriate D-Rules.

$$D\left(e^{\left(\frac{x+1}{x-1}\right)}\right) =$$

$$e^{\left(\frac{x+1}{x-1}\right)} \cdot \left[\frac{D(x+1)(x-1) - D(x-1)(x+1)}{(x-1)^2}\right]$$

$$'' \cdot \frac{(x-1)-(x+1)}{(x-1)^2}$$

$$'' \cdot \frac{x-1-x-1}{(x-1)^2}$$

$$'' \cdot \frac{-2}{(x-1)^2}$$

Red Problem 8. Compute the derivative using the appropriate D-Rules.

$$D(\ln(x^4 - x^2 + 1)) =$$

$$= \frac{1}{x^4 - x^2 + 1} D[x^4 - x^2 + 1] =$$

$$= \frac{4x^3 - 2x}{x^4 - x^2 + 1}$$

Red Problem 9. Compute the derivative using the appropriate D-Rules.

$D(\cos(\ln(x))) =$

$$-\sin(\ln x) \cdot D[\ln x] =$$

$$-\sin(\ln x) \cdot \frac{1}{x} =$$

$$\frac{-\sin(\ln x)}{x}$$

Red Problem 10. Compute the derivative using the appropriate D-Rules.

$D(\sqrt{\ln(x)}) =$

$$D\left[(\ln x)^{\frac{1}{2}}\right] = \frac{1}{2}(\ln x)^{-\frac{1}{2}} D[\ln x]$$

$$= \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$$

$$= \frac{1}{2x\sqrt{\ln x}}$$

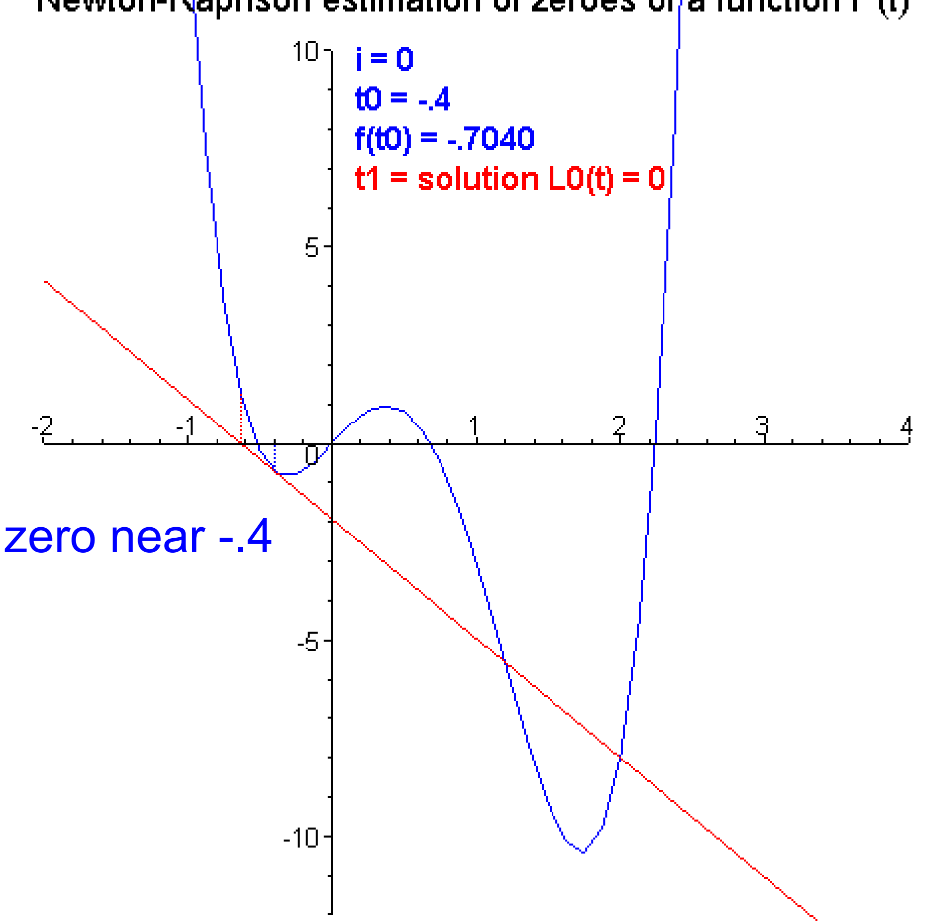
Problem 11. Suppose that the function f(t) describes the motion of some object or particle. Find the velocity and acceleration functions for this motion. How many times does the motion of this object have zero velocity? Explain by graphing f(t) and describing the motion.

$$f(t) = t^6 - 6t^4 + 7t^2$$

Problem 12. Use the Newton-Raphson method to find the x-coordinates of the bump points (peaks, valleys, and plateaus) of f(x), carefully showing all your work.

$f(x) = x^5 - 3x^4 + 2x^2 + 1$ or $f(t) = t^5 - 3t^4 + 2t^2 + 1$

Newton-Raphson estimation of zeroes of a function f '(t)



Estimating zero near -.4

$f'(t) = 5t^4 - 12t^3 + 4t,\ f''(t) = 20t^3 - 36t^2 + 4$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

$$f'(t) = 5t^4 - 12t^3 + 4t,\ f''(t) = 20t^3 - 36t^2 + 4$$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

Iteration 0

$$t_0 = -0.4,\ \ L_0(t) = -3.040\,t - 1.9200$$

$$\textit{Solve } L_0(t) = 0, \ \textit{or equivalently},\ t_1 = t_0 - \frac{f'(t_0)}{f''(t_0)}$$

$$t_1 = -0.6315789474,\ f'(t_1) = 1.292439438$$

Iteration 1

$$t_1 = -0.6315789474,\ \ L_1(t) = -15.39874617\,t - 8.433084459$$

$$\textit{Solve } L_1(t) = 0, \ \textit{or equivalently},\ t_2 = t_1 - \frac{f'(t_1)}{f''(t_1)}$$

$$t_2 = -0.5476474751,\ f'(t_2) = 0.230153833$$

Iteration 2

$$t_2 = \text{-}0.5476474751,\ L_2(t) = -10.08202330\,t - 5.291240771$$

$$\textit{Solve } L_2(t) = 0,\ \textit{or equivalently},\ t_3 = t_2 - \frac{f'(t_2)}{f''(t_2)}$$

$$t_3 = \text{-}0.5248193357,\ f'(t_3) = 0.014691251$$

Iteration 3

$$t_3 = \text{-}0.5248193357,\ L_3(t) = -8.80674786\,t - 4.607260311$$

$$\textit{Solve } L_3(t) = 0,\ \textit{or equivalently},\ t_4 = t_3 - \frac{f'(t_3)}{f''(t_3)}$$

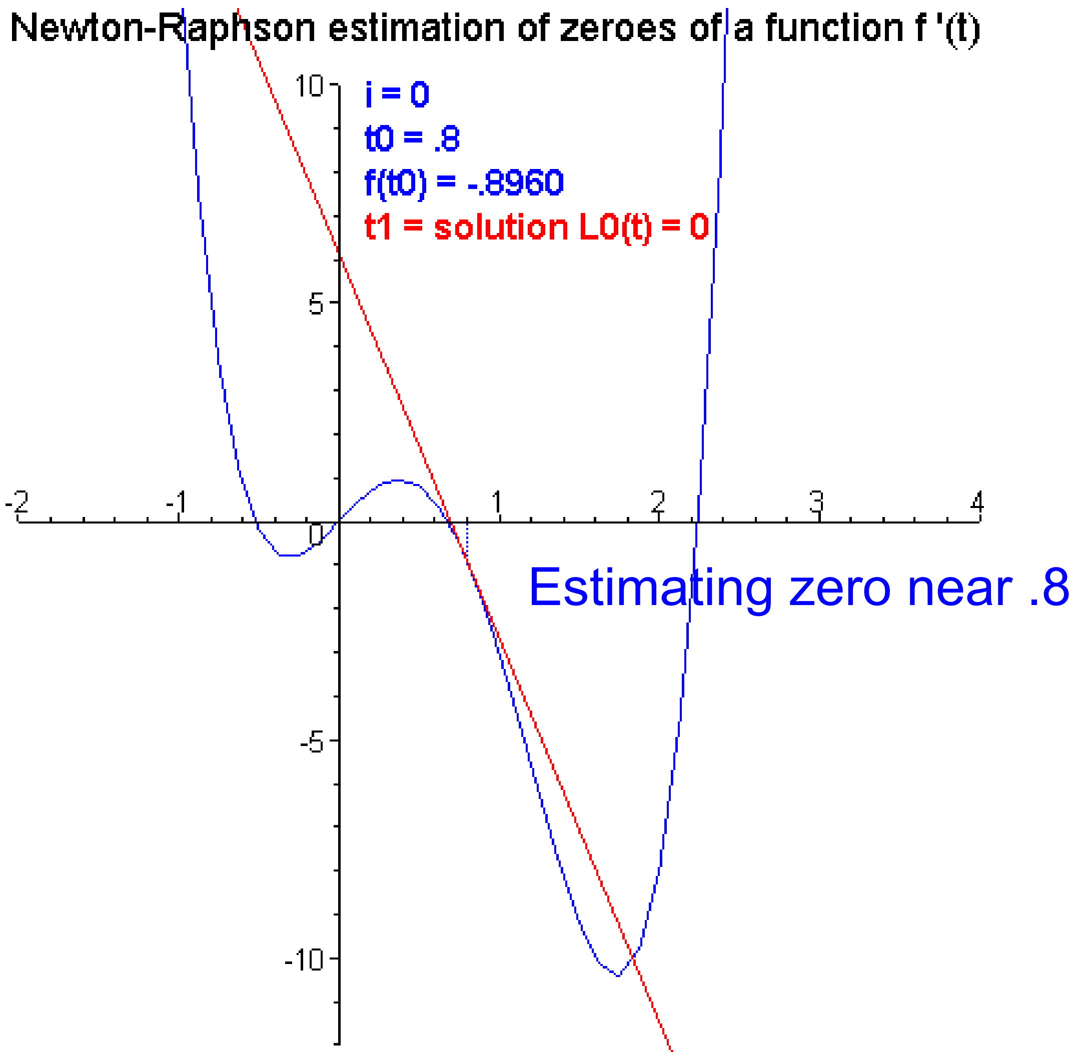
$$t_4 = \text{-}0.5231511546,\ f'(t_4) = 0.000075470$$

Iteration 4

$$t_4 = \text{-}0.5231511546,\ L_4(t) = -8.71633147\,t - 4.559883402$$

$$\textit{Solve } L_4(t) = 0,\ \textit{or equivalently},\ t_5 = t_4 - \frac{f'(t_4)}{f''(t_4)}$$

$$t_5 = \text{-}0.5231424961,\ f'(t_5) = 0.1\ 10^{-8}$$



$f'(t) = 5t^4 - 12t^3 + 4t, f''(t) = 20t^3 - 36t^2 + 4$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

$$f'(t) = 5t^4 - 12t^3 + 4t, \; f''(t) = 20t^3 - 36t^2 + 4$$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

Iteration 0

$$t_0 = 0.8, \; L_0(t) = -8.800\,t + 6.1440$$

$$\textit{Solve } L_0(t) = 0, \; \textit{or equivalently}, \; t_1 = t_0 - \frac{f'(t_0)}{f''(t_0)}$$

$$t_1 = 0.6981818182, \; f'(t_1) = -0.103207450$$

Iteration 1

$$t_1 = 0.6981818182, \; L_1(t) = -6.74179847\,t + 4.603793664$$

$$\textit{Solve } L_1(t) = 0, \; \textit{or equivalently}, \; t_2 = t_1 - \frac{f'(t_1)}{f''(t_1)}$$

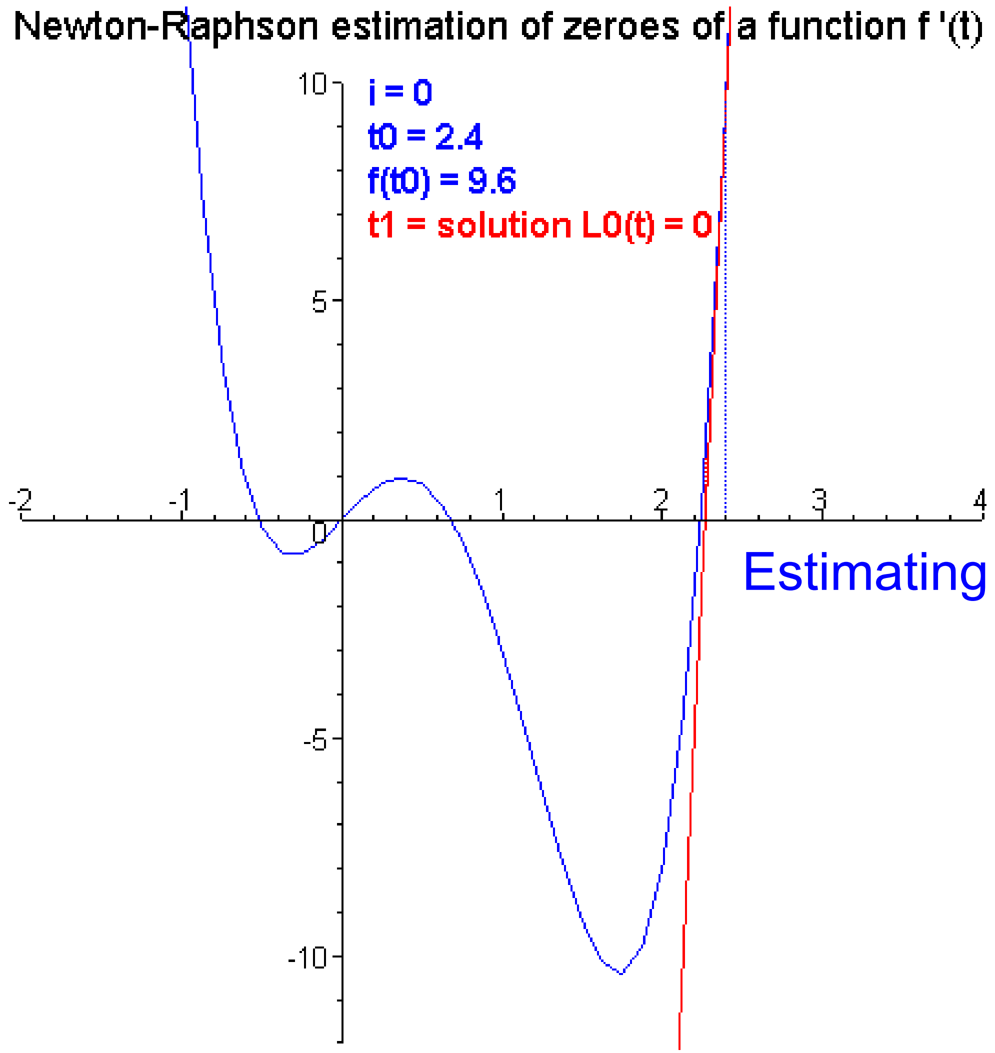
$$t_2 = 0.6828732251, \; f'(t_2) = -0.002470011$$

Iteration 2

$$t_2 = 0.6828732251, \; L_2(t) = -6.41867825\,t + 4.380673506$$

$$\textit{Solve } L_2(t) = 0, \; \textit{or equivalently}, \; t_3 = t_2 - \frac{f'(t_2)}{f''(t_2)}$$

$$t_3 = 0.6824884089, \; f'(t_3) = -0.1568\;10^{-5}$$



Estimating zero near 2.4

$$f'(t) = 5t^4 - 12t^3 + 4t, \; f''(t) = 20t^3 - 36t^2 + 4$$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

$$f'(t) = 5t^4 - 12t^3 + 4t, \quad f''(t) = 20t^3 - 36t^2 + 4$$

Estimate zeroes of $f'(t)$ *using Newton-Raphson Method:*

Iteration 0

$$t_0 = 2.4, \quad L_0(t) = 73.120\, t - 165.8880$$

$$\text{Solve } L_0(t) = 0, \text{ or equivalently, } t_1 = t_0 - \frac{f'(t_0)}{f''(t_0)}$$

$$t_1 = 2.268708972, \quad f'(t_1) = 1.409319488$$

Iteration 1

$$t_1 = 2.268708972, \quad L_1(t) = 52.2492802\, t - 117.1290913$$

$$\text{Solve } L_1(t) = 0, \text{ or equivalently, } t_2 = t_1 - \frac{f'(t_1)}{f''(t_1)}$$

$$t_2 = 2.241735979, \quad f'(t_2) = 0.052267516$$

Iteration 2

$$t_2 = 2.241735979, \quad L_2(t) = 48.3978248\, t - 108.4428777$$

$$\text{Solve } L_2(t) = 0, \text{ or equivalently, } t_3 = t_2 - \frac{f'(t_2)}{f''(t_2)}$$

$$t_3 = 2.240656024, \quad f'(t_3) = 0.000081596$$