# Forward Derivatives

$D[1] = ?\ 0$

$D[x] = ?\ 1$

$D[x^2] = ?\ 2x$

$D[x^3] = ?\ 3x^2$

$\vdots$

$D[x^n] = ?\ nx^{n-1}$

$D[a] = ?\ 0$

$D[ax] = ?\ a$

$D[ax^2] = ?\ 2ax$

$D[ax^3] = ?\ 3ax^2$

$\vdots$

$D[ax^n] = ?\ nax^{n-1}$

# Reverse Derivatives

| | | | |
|---|---|---|---|
| $1$ | $D[\,?\,] = 0$ | $C$ | $D[\,?\,] = 0$ |
| $x$ | $D[\,?\,] = 1$ | $ax$ | $D[\,?\,] = a$ |
| $\frac{x^2}{2}$ | $D[\,?\,] = x$ | $\frac{ax^2}{2}$ | $D[\,?\,] = ax$ |
| $\frac{x^3}{3}$ | $D[\,?\,] = x^2$ | $\frac{ax^3}{3}$ | $D[\,?\,] = ax^2$ |
| $\frac{x^4}{4}$ | $D[\,?\,] = x^3$ | $\frac{ax^4}{4}$ | $D[\,?\,] = ax^3$ |
| | . | | . |
| | . | | . |
| | . | | . |
| $\frac{x^{n+1}}{n+1}$ | $D[\,?\,] = x^n$ | $\frac{ax^{n+1}}{n+1}$ | $D[\,?\,] = ax^n$ |

## Forward Derivatives

$D[e^x] = ?\ e^x$

$D[e^{2x}] = ?\ 2e^{2x}$

$D[e^{3x}] = ?\ 3e^{3x}$

$D[e^{ax}] = ?\ ae^{ax}$

## Reverse Derivatives

$e^x \quad D[\ ?\ ] = e^x$

$\frac{e^{2x}}{2} \quad D[\ ?\ ] = e^{2x}$

$\frac{e^{3x}}{3} \quad D[\ ?\ ] = e^{3x}$

$\frac{e^{ax}}{a} \quad D[\ ?\ ] = e^{ax}$

| Forward Derivatives | Reverse Derivatives |
| --- | --- |
| | No rule to reverse? |
| $D[\ln x] = ?$ $\frac{1}{x}$ | $D[?] = \ln x$ |
| $D[\ln 2x] = ?$ $\frac{D[2x]}{2x} = \frac{1}{x}$ | $D[?] = \ln 2x$ |
| $D[\ln 3x] = ?$ $\frac{1}{x}$ | $D[?] = \ln 3x$ |
| $D[\ln ax] = ?$ $\frac{1}{x}$ | $D[?] = \ln ax$ |

| Forward Derivatives | Reverse Derivatives |
| --- | --- |
| $D[\sin x] = ? \cos x$ | $D[?] = \sin x$ $-\cos x$ |
| $D[\sin 2x] = ? 2\cos 2x$ | $D[?] = \sin 2x$ $-\frac{\cos 2x}{2}$ |
| $D[\sin 3x] = ? 3\cos 3x$ | $D[?] = \sin 3x$ $-\frac{\cos 3x}{3}$ |
| $D[\sin ax] = ? a\cos ax$ | $D[?] = \sin ax$ $-\frac{\cos ax}{a}$ |

## Forward Derivatives

$D[\cos x] = ? -\sin x$

$D[\cos 2x] = ? -2\sin 2x$

$D[\cos 3x] = ? -3\sin 3x$

$D[\cos ax] = ? -a\sin ax$

## Reverse Derivatives

$D[\ ? \ ] = \cos x$ $\quad \sin x$

$D[\ ? \ ] = 2\cos 2x$ $\quad \frac{\sin 2x}{2}$

$D[\ ? \ ] = 3\cos 3x$ $\quad \frac{\sin 3x}{3}$

$D[\ ? \ ] = \cos ax$ $\quad \frac{\sin ax}{a}$

# More Reverse Derivatives

$\ln x \qquad D[\,?\,] = \frac{1}{x^1} = x^{-1}$

$\frac{x^{-1}}{-1} \qquad D[\,?\,] = \frac{1}{x^2} = x^{-2}$

$\frac{x^{-2}}{-2} \qquad D[\,?\,] = \frac{1}{x^3} = x^{-3}$

$\frac{x^{-n+1}}{-n+1} \qquad D[\,?\,] = \frac{1}{x^n} = x^{-n}$

$n \neq 1$

$\ln x \qquad D[\,?\,] = \frac{1}{x}$

$\ln x^2 = 2\ln x \qquad D[\,?\,] = \frac{2}{x}$

$\ln x^3 = 3\ln x \qquad D[\,?\,] = \frac{3}{x}$

$\ln x^n = n \cdot \ln x \qquad D[\,?\,] = \frac{n}{x}$

# Reverse Derivative Rules

Given a function f(x), we want to solve D[ h(x) ] = f(x) for a reverse derivative function h(x).

If D[ h(x) ] = 0, then h(x) = C (any constant number)

If D[ h(x) ] = a, then h(x) = ax + C

If D[ h(x) ] = x, then h(x) = $\frac{x^2}{2}$ + C

If D[ h(x) ] = $x^n$, then h(x) = $\frac{x^{n+1}}{n+1}$ + C , n ≠ -1

If D[ h(x) ] = $\frac{1}{x}$ , then h(x) = ln x + C

# More Reverse Derivative Rules

Given a function f(x), we want to solve D[ h(x) ] = f(x)
for a reverse derivative function h(x).

If $D[ h(x) ] = e^{ax}$, then $h(x) = \frac{e^{ax}}{a} + C$

If $D[ h(x) ] = \sin(ax)$, then $h(x) = -\frac{\cos(ax)}{a} + C$

If $D[ h(x) ] = \cos(ax)$, then $h(x) = \frac{\sin(ax)}{a} + C$