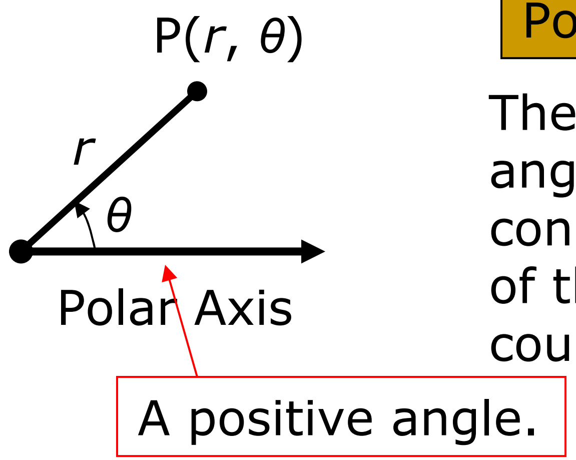
# Polar Coordinates

Polar and Cartesian Coordinates
The Equation of a Line
The Equation of a Circle
Polar Curves
Areas of Polar Domains

# Polar Coordinates

To define the Polar Coordinates of a plane we need first to fix a point which will be called the Pole (or the origin) and a half-line starting from the pole. This half-line is called the Polar Axis.



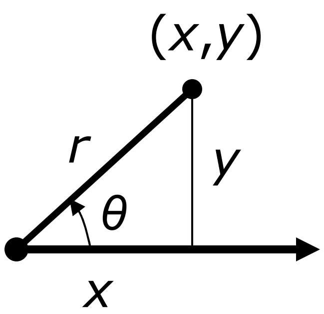

**Polar Angles**

The Polar Angle $\theta$ of a point P, P ≠ pole, is the angle between the Polar Axis and the line connecting the point P to the pole. Positive values of the angle indicate angles measured in the counterclockwise direction from the Polar Axis.

**Polar Coordinates**

The Polar Coordinates $(r,\theta)$ of the point P, P ≠ pole, consist of the distance $r$ of the point P from the Pole and of the Polar Angle θ of the point $P$. Every $(0, \theta)$ represents the pole.

# Polar and Cartesian Coordinates




From the right angle triangle in the picture one immediately gets the following correspondence between the Cartesian Coordinates $(x,y)$ and the Polar Coordinates $(r,\theta)$ assuming the Pole of the Polar Coordinates is the Origin of the Cartesian Coordinates and the Polar Axis is the positive $x$-axis.

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

$$r^2 = x^2 + y^2$$

$$\tan(\theta) = y/x$$

Using these equations one can easily switch between the Cartesian and the Polar Coordinates.

# The Equation of a Line

The Cartesian equation of a general line is of the form

$$ax + by + c = 0$$

If the line in question passes through the origin, $c = 0$.
In this case the Polar Coordinate Equation for the line is

$$\tan(\theta) = -a/b.$$

If the line does not pass through the origin, one needs to substitute $x = r\cos(\theta)$ and $y = r\sin(\theta)$ to the equation of the line. One gets

$$a\, r\cos(\theta) + b\, r\sin(\theta) = c.$$

This yields $r = -c/(a\cos(\theta) + b\sin(\theta))$.

One concludes that the equation of a general line in the Polar Coordinates is rather complicated.

# The Equation of a Circle

The Cartesian Equation of a Circle with radius $r_0$ and with center at the origin is

$$x^2 + y^2 = r_0^2.$$

In the Polar Coordinates this equation becomes remarkably simple:

$$r = r_0.$$

# Polar Curves

Definition

A Polar Curve consists of all the points $(r,\theta)$ satisfying a given equation

$$F(r,\theta) = 0.$$

Often one can solve r form the equation and represent the polar curve in the form

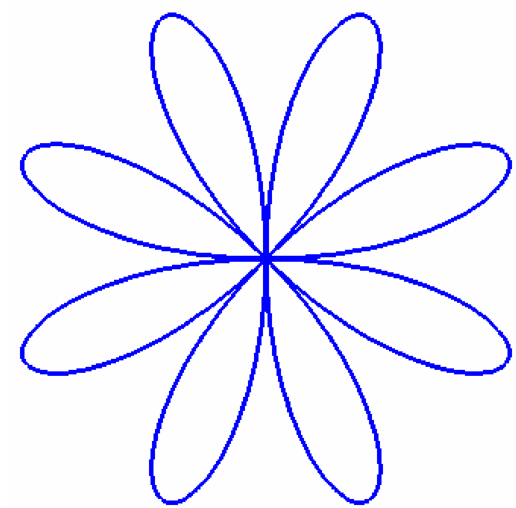
$$r = f(\theta)$$

Plotting Polar Curves by Maple

Use the option "coords = polar" in the plot command to plot curves defined in the polar coordinates. For the Maple command

```
plot([sin(4*t),t,t=0..2*Pi],coords=polar);
```

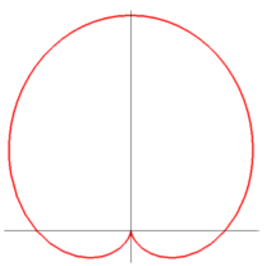
produces the following plot.



# Areas of Polar Domains

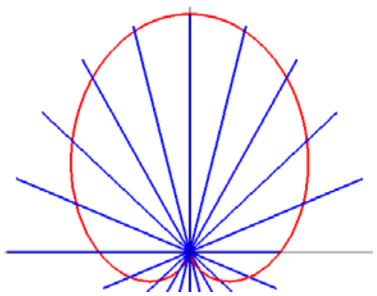
To compute the area of a polar domain, i.e., the area inside the graph of a polar curve, one repeats the same arguments as the ones used in the Cartesian case.

One divides the domain in question by half-line emanating from the origin and forms a Riemann sum approximation for the area of the domain in question.



Polar Curve
$r = 1 + \sin(\theta)$

Divide a polar domain into several sections whose area will be approximated by areas of circular sections.



Approximate the areas of the sections by the areas of sections of circles.

# Areas of Polar Domains

**Formula**

The area enclosed by the graph of the polar curve

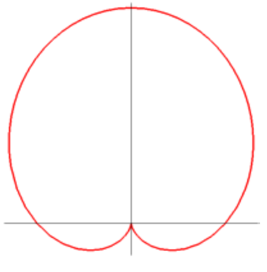
$r = f(\theta)$

and by the polar lines $\theta = a$ and $\theta = b$ is given by

$$A = \int_a^b \frac{1}{2}\left(f(\theta)\right)^2 d\theta \quad \text{or} \quad A = \int_a^b \frac{1}{2} r^2 \, d\theta$$

**Example**

The area of the domain enclosed by the Polar Curve $r = 1 + \sin(\theta)$ is computed as follows.



$$A = \frac{1}{2}\int_0^{2\pi} \left(1 + \sin(\theta)\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(1 + 2\sin(\theta) + \sin^2(\theta)\right) d\theta = \frac{3\pi}{2}$$