Polar Notes

Coordinate Conversion

Polar to Rectangular

$x = r\cos\theta$

$y = r\sin\theta$

Rectangular to Polar

$r^2 = x^2 + y^2$

$\tan\theta = y/x$

Slope in Polar Form

If f is a differentiable function of θ, then the slope of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

provided that $dx/d\theta \neq 0$ at (r, θ).

Tangent Lines at the Pole

If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the graph of $r = f(\theta)$.

Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \; d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \; d\theta$$

Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \quad \theta \quad \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} \; d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \; d\theta$$

Area of a Surface of Revolution

Let f be a function whose derivative is continuous on an interval $\alpha \quad \theta \quad \beta$. The area of the surface formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows

1. $S = 2\pi \int_{\alpha}^{\beta} f(\theta)\sin\theta\sqrt{[f(\theta)]^2 + [f'(\theta)]^2} \; d\theta$ — About the polar axis

2. $S = 2\pi \int_{\alpha}^{\beta} f(\theta)\cos\theta\sqrt{[f(\theta)]^2 + [f'(\theta)]^2} \; d\theta$ — About the line $\theta = \frac{\pi}{2}$

Useful Integrals

(a is a constant)

$$\int \cos^2(a\theta)\ d\theta = \frac{1}{2}\theta + \frac{1}{4a}\sin(2a\theta) + C$$

$$\int \sin^2(a\theta)\ d\theta = \frac{1}{2}\theta - \frac{1}{4a}\sin(2a\theta) + C$$

$$\int \sqrt{1+\cos(a\theta)}\ d\theta = \frac{2\sqrt{2}}{a}\sin\left(\frac{a\theta}{2}\right) + C, \qquad \cos\left(\frac{a\theta}{2}\right) \geq 0$$

$$\int \sqrt{1-\cos(a\theta)}\ d\theta = -\frac{2\sqrt{2}}{a}\cos\left(\frac{a\theta}{2}\right) + C, \qquad \sin\left(\frac{a\theta}{2}\right) \geq 0$$

$$\int \sqrt{1+\sin(a\theta)}\ d\theta = -\frac{2}{a}\sqrt{1+\sin(a\theta)} + C, \qquad \cos(a\theta) \geq 0$$

$$\int \sqrt{1-\sin(a\theta)}\ d\theta = \frac{2}{a}\sqrt{1+\sin(a\theta)} + C, \qquad \cos(a\theta) \geq 0$$

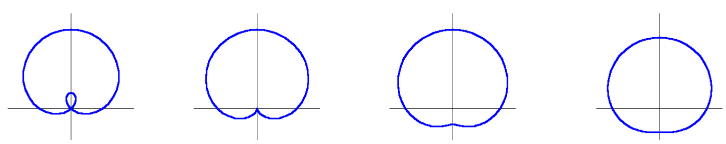
Special Polar Graphs

Limaçons

$r = a \pm b\cos\theta$

$r = a \pm b\sin\theta$

$(a > 0, b > 0)$



$\frac{a}{b} < 1$	$\frac{a}{b} = 1$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} \geq 2$
Limaçon with inner loop	Cardioid (heart-shaped)	Dimpled limaçon	Convex limaçon

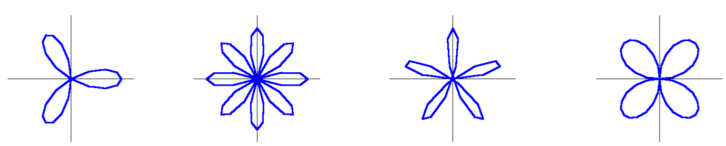
Rose Curves

n petals if n is odd

$2n$ petals if n is even

$(n \geq 2)$

a: Distance from polar axis to the tip of the petal



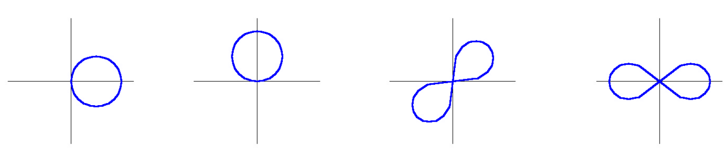
$r = a\cos n\theta$	$r = a\cos n\theta$	$r = a\sin n\theta$	$r = a\sin n\theta$
$(n = 3)$	$(n = 4)$	$(n = 5)$	$(n = 2)$

Circles

a: Diameter of the circle

Lemniscates

a: Distance from the pole to the tip of a loop



$r = a\cos\theta$	$r = a\sin\theta$	$r^2 = a^2\sin 2\theta$	$r^2 = a^2\cos 2\theta$
Circle	Circle	Lemniscate	Lemniscate