

Arc Length 3.1 (Jon)

$$y = \frac{x^3}{6} + \frac{1}{2x}, \quad x=1..2$$

$$f'(x) = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$(f'(x))^2 = \frac{x^4}{4} - 2\left(\frac{1}{2}x^2\right)\left(\frac{1}{2x^2}\right) + \frac{1}{4x^4}$$

$$\int_1^2 \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}$$

$$\int_1^2 \sqrt{\frac{x^4}{4} + \frac{1}{4x^4} + \frac{1}{2}}$$

$$\int_1^2 \sqrt{\frac{(x^4+1)^2}{4x^4}} = \int_1^2 \frac{x^4+1}{2x^2} = \frac{1}{2} \int_1^2 x^2 + x^{-2}$$

$$\begin{aligned} & \rightarrow \frac{1}{2} \left(\frac{x^3}{3} - \frac{1}{x} \right) \Big|_1^2 \\ & = \frac{1}{2} \left[\left(\frac{8}{3} - \frac{1}{2} \right) - \left(\frac{1}{3} - 1 \right) \right] \\ & = \frac{17}{12} \end{aligned}$$

$$x = \frac{2(y-1)^{3/2}}{3}$$

$$f'(y) = \sqrt{y-1}$$

$$\left. \begin{array}{l} 5 \\ 2 \end{array} \right\} \sqrt{1 + (\sqrt{y-1})^2}$$

$$\left. \begin{array}{l} 5 \\ 2 \end{array} \right\} \sqrt{1 + y - 1}$$

$$\left. \begin{array}{l} 5 \\ 2 \end{array} \right\} \sqrt{y}$$

$$\frac{2y^{2/3}}{3} \Big|_2^5$$

$$\frac{2(5)^{2/3}}{3} - \frac{2(2)^{2/3}}{3}$$

$$\frac{10\sqrt[3]{5}}{3} - \frac{4\sqrt[3]{2}}{3}$$

$$\approx 5.568$$

Problem 3.2
Holla

$$y = \ln(\sin x)$$

$$y' = \frac{\cos x}{\sin x}$$

$$\int_{\pi/9}^{\pi/3} \sqrt{1 + \cot^2 x} \, dx$$

$$\int_{\pi/9}^{\pi/3} \csc x \, dx \left(\frac{\csc x + \cot x}{\csc x + \cot x} \right)$$

$$\frac{\csc^2 x + \cot x \csc x}{\csc x + \cot x}$$

$$u = \csc x + \cot x$$

$$du = -\csc^2 x - \cot x \csc x$$

$$\int \frac{1}{u} \, du$$

$$-\ln |\csc x + \cot x| \Big|_{\pi/9}^{\pi/3}$$

$$-\ln \left| \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3} \right| + \ln |2 + \sqrt{3}|$$

$$\ln \left| \frac{2 + \sqrt{3}}{\sqrt{3}} \right|$$

Arc length
3.3
2.12

Arc length 3.6 π my

$$x = e^t - t \quad y = 4te^{(t/2)} \quad t = -8 \dots 3$$

$$x'(t) = e^t - 1$$

$$y'(t) = 2e^{.5t}$$

$$(x'(t))^2 = (e^t - 1)^2$$

$$(y'(t))^2 = 4e^t$$

$$= e^{2t} - 2e^t + 1$$

$$\int_{-8}^3 \sqrt{e^{2t} - 2e^t + 1 + 4e^t} dt$$

$$\int_{-8}^3 \sqrt{e^{2t} + 1 + 2e^t} dt$$

$$\int_{-8}^3 \sqrt{(e^t + 1)^2} dt$$

$$\int_{-8}^3 e^t + 1 dt$$

$$(e^t + t) \Big|_{-8}^3$$

$$(e^3 + 3) - (e^{-8} - 8)$$

$$\approx 31.08$$

3.5- $x = e^t \cos(t)$ $y = e^t \sin(t)$ $t = 0 \dots \pi$ Alex S.

$$x'(t) = e^t \cos t - e^t \sin t \quad [x'(t)]^2 = e^{2t} \cos^2 t - 2e^t \cos t \sin t + e^{2t} \sin^2 t$$

$$y'(t) = e^t \sin t + e^t \cos t \quad [y'(t)]^2 = e^{2t} \sin^2 t + 2e^t \cos t \sin t + e^{2t} \cos^2 t$$

$$L = \int_0^\pi \sqrt{2e^{2t} (\cos^2 t + \sin^2 t)} dt$$

$$= \int_0^\pi \sqrt{2e^{2t}} dt$$

$$= \int_0^\pi \sqrt{2} e^t dt$$

$$= \sqrt{2} e^t \Big|_0^\pi$$

$$= \sqrt{2} (e^\pi - e^0)$$

$$= \sqrt{2} e^\pi - \sqrt{2}$$

$$\approx 31.31 \text{ units}$$

II ^{3.9} $\int \sin(x)^5 (\cos(x))^2 dx$

David Leander

$$= \int (1 - \cos^2 x)^2 \cdot \cos^2 x \cdot \sin x dx$$

$$= \int (1 - 2\cos^2 x + \cos^4 x) \cdot \cos^2 x \cdot \sin x dx$$

$$= \int \cos^2 x \sin x dx - \int 2\cos^2 x (\cos^2 x \sin x) + \int \cos^4 x (\cos^2 x \sin x) dx$$

$$= -\frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C$$

II 3.5
Amy

$$\int \cos^3(x) \sin^4(x) dx$$

$$\int \cos^4(x) \cos(x) \sin^4(x) dx$$

$$\int (1 - \sin^2(x))^2 \cos(x) \sin^4(x) dx$$

$$\int (1 - u^2)^2 \cancel{\cos(x)} u^4 \frac{du}{\cancel{\cos(x)}}$$

$$\int (1 - 2u^2 + u^4) u^4 du$$

$$\int u^4 - 2u^6 + u^8 du$$

$$\frac{1}{5} u^5 - \frac{2}{7} u^7 + \frac{1}{9} u^9 + C$$

$$\frac{1}{5} \sin^5 x - \frac{2}{7} \sin^7 x + \frac{1}{9} \sin^9 x + C$$

$$u = \sin(x) \\ du = \cos x dx$$

II 3.8

David G.

$$\tan x = u$$

$$dx = \frac{du}{\sec^2 x}$$

$$\int \tan^3 x dx$$

$$\int \sec^2 x \tan x - \tan x dx$$

$$\int u du - \int \tan x dx$$

$$\frac{\tan^2 x}{2} + \ln|\cos x| + C$$

II 3.9 Max Z.

$$\int \tan^6 x \sec^4 x dx \rightarrow \int \underbrace{\tan^6 x \sec^2 x}_u \frac{d \tan x dx}{\underbrace{dv}_{v = \tan x}}$$

$$du = 6 \tan^5 x \sec^4 x + 2 \tan^6 x \sec^2 x$$

$$\int = \tan^7 x \sec^2 x - 6 \int \tan^6 x \sec^4 x dx - 2 \int \tan^8 x dx$$

$$\int = \tan^7 x \sec^2 x - \frac{2}{9} \tan^9 x$$

$$= \frac{1}{9} \tan^7 x \sec^2 x - \frac{2}{63} \tan^9 x + C$$

II 3.9 Jan

$$\int \tan^6 x \sec^4 x \, dx$$

$$= \int \tan^6 x \sec^2 x \cdot \sec^2 x \, dx$$

$$= \int \tan^6 x (\tan^2 x + 1) \sec^2 x \, dx$$

$$= \int (\tan^8 x + \tan^6 x) \sec^2 x \, dx$$

$$= \frac{\tan^9 x}{9} + \frac{\tan^7 x}{7} + C$$

II 3.10 Alex Server

$$\int \sec^7 x \tan^5 x dx$$

$$\text{let } u = \sec x$$

$$du = \sec x \tan x dx$$

$$= \int \sec^6 x \tan^4 x \sec x \tan x dx$$

$$= \int \sec^6 x (\sec^2 x - 1)^2 \sec x \tan x dx$$

$$= \int \sec^6 x (\sec^4 x - 2\sec^2 x + 1) \sec x \tan x dx$$

$$= \int (\sec^{10} x - 2\sec^8 x + \sec^6 x) \sec x \tan x dx$$

$$= \int \sec^{10} x \sec x \tan x dx - 2 \int \sec^8 x \sec x \tan x dx + \int \sec^6 x \sec x \tan x dx$$

$$= \frac{1}{11} \sec^{11} x - \frac{2}{9} \sec^9 x + \frac{1}{7} \sec^7 x + C$$