Arc Length 3.1 (Jon)

$y = \frac{x^3}{6} + \frac{1}{2x}, \; x = 1..2$

$f'(x) = \frac{1}{2}x^2 - \frac{1}{2x^2}$

$(f'(x))^2 = \frac{x^4}{4} - 2\left(\frac{1}{2}x^2\right)\left(\frac{1}{2x^2}\right) + \frac{1}{4x^4}$

$\int_1^2 \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}$

$\int_1^2 \sqrt{\frac{x^4}{4} + \frac{1}{4x^4} + \frac{1}{2}}$

$\int_1^2 \sqrt{\frac{(x^4+1)^2}{4x^4}} = \int_1^2 \frac{x^4+1}{2x^2} = \frac{1}{2}\int_1^2 x^2 + x^{-2}$

$\frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{x}\right)\Big|_1^2$

$\frac{1}{2}\left[\left(\frac{8}{3} - \frac{1}{2}\right) - \left(\frac{1}{3} - 1\right)\right]$

$= \boxed{\frac{17}{12}}$

Problem 3.2
Holl A

$$x = \frac{2(y-1)^{3/2}}{3}$$

$$f'(y) = \sqrt{y-1}$$

$$\int_2^5 \sqrt{1+(\sqrt{y-1})^2}$$

$$\int_2^5 \sqrt{1+y-1}$$

$$\int_2^5 \sqrt{y}$$

$$\left.\frac{2y^{2/3}}{3}\right|_2^5$$

$$\frac{2(5)^{2/3}}{3} - \frac{2(2)^{2/3}}{3}$$

$$\frac{10\sqrt{5}}{3} - \frac{4\sqrt{2}}{3}$$

$$\approx 5.568$$

$$y = \ln(\sin(x))$$

$$y' = \frac{\cos x}{\sin x}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1+\cot^2 x}\ dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \csc x\ dx \left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\csc^2 + \cot \csc}{\csc x + \cot x}$$

$u = \csc + \cot x$

$du = -\csc^2 x\ \cot \csc$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} -\frac{1}{u}\ du$$

$$-\ln|\csc x + \cot x|\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$-\ln\left|\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\right| + \ln|2+\sqrt{3}|$$

$$\ln\left|\frac{2+\sqrt{3}}{\sqrt{3}}\right|$$

Arc Length

3.3

Eliz

Arc Length 3.6 Jimmy

$x = e^t - t \quad y = 4e^{(t/2)} \quad t = -8..3$

$x'(t) = e^t - 1 \qquad y'(t) = 2e^{.5t}$

$(x'(t))^2 = (e^t - 1)^2 \qquad (y'(t))^2 = 4e^t$

$= e^{2t} - 2e^t + 1$

$$\int_{-8}^{3} \sqrt{e^{2t} - 2e^t + 1 + 4e^t}\, dt$$

$$\int_{-8}^{3} \sqrt{e^{2t} + 1 + 2e^t}\, dt$$

$$\int_{-8}^{3} \sqrt{(e^t + 1)^2}\, dt$$

$$\int_{-8}^{3} e^t + 1\, dt$$

$$(e^t + t)\Big|_{-8}^{3}$$

$$(e^3 + 3) - (e^{-8} - 8)$$

$$\approx 31.08$$

3.5 $x = e^{t}\cos(t)$ $y = e^{t}\sin t$ $t = 0 \ldots \pi$ Alex S.

$x'(t) = e^{t}\cos t - e^{t}\sin t$ $[x'(t)]^{2} = e^{2t}\cos^{2}t - 2e^{t}\cos t\sin t + e^{2t}\sin^{2}t$

$y'(t) = e^{t}\sin t + e^{t}\cos t$ $[y'(t)]^{2} = e^{2t}\sin^{2}t + 2e^{t}\cos t\sin t + e^{2t}\cos^{2}t$

$$L = \int_{0}^{\pi} \sqrt{2e^{t}(\cos^{2}t + \sin^{2}t)}\, dt$$

$$= \int_{0}^{\pi} \sqrt{2e^{2t}}\, dt$$

$$= \int_{0}^{\pi} \sqrt{2}\, e^{t}\, dt$$

$$= \sqrt{2}\, e^{t} \Big|_{0}^{\pi}$$

$$= \sqrt{2}(e^{\pi} - e^{0})$$

$$= \sqrt{2}\, e^{\pi} - \sqrt{2}$$

≈ 31.31 units

II 3.4 $\int \sin(x)^5 \cos(x)^2\,dx$

David Lennaev

$$= \int (1-\cos^2 x)^2 \cdot \cos^2 x \cdot \sin x\,dx$$

$$= \int (1-2\cos^2 x+\cos^4 x)\cdot \cos^2 x \cdot \sin x\,dx$$

$$= \int \cos^2 x \sin x\,dx - \int 2\cos^2 x(\cos^2 x \sin x) + \int \cos^4 x(\cos^2 x \sin x\,d$$

$$\boxed{= \frac{-\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C}$$

II 3.5

Amy

$$\int \cos^5(x)\sin^4(x)\,dx$$

$$\int \cos^4(x)\cos(x)\sin^4(x)\,dx$$

$$\int (1-\sin^2(x))^2\cos(x)\sin^4(x)\,dx$$

$u=\sin(x)$

$du=\cos x\,dx$

$$\int (1-u^2)^2\,\cancel{\cos(x)}\,u^4\,\frac{du}{\cancel{\cos(x)}}$$

$$\int (1-2u^2+u^4)\,u^4\,du$$

$$\int u^4-2u^6+u^8\,du$$

$$\frac{1}{5}u^5-\frac{2}{7}u^7+\frac{1}{9}u^9+c$$

$$\frac{1}{5}\sin^5 x-\frac{2}{7}\sin^7 x+\frac{1}{9}\sin^9 x+c$$

II 3.8

David G.

$$\int \tan^3 x \, dx$$

$\tan x = u$

$dx = \dfrac{du}{\sec^2 x}$

$$\int \sec^2 x \tan x - \tan x \, dx$$

$$\int u \, du - \int \tan x \, dx$$

$$\frac{\tan^2 x}{2} + \ln|\cos x| + c$$

II 3.9 Max Z.

$$\int \tan^6 x \sec^4 x \, dx \rightarrow \int \underbrace{\tan^6 x \sec^2 x}_{u} \, \underbrace{d\tan x}_{dv} \, dx$$

$v = \tan x$

$du = 6\tan^5 x \sec^4 x + 2\tan^7 x \sec^2 x$

$\int = \tan^7 x \sec^2 x - 6\int \tan^6 x \sec^4 x \, dx - 2\int \tan^8 x \, d\tan x$

$7\int = \tan^7 x \sec^2 x - \frac{2}{9}\tan^9 x$

$= \frac{1}{7}\tan^7 x \sec^2 x - \frac{2}{63}\tan^9 x + C$

II 3.9 Jon

$$\int \tan^6 x \sec^4 x \, dx$$

$$= \int \underbrace{\tan^6 x \sec^2 x}_{u} \; \underbrace{\sec^2 x \, dx}_{du}$$

$$= \int \tan^6 x (\tan^2 x + 1) \sec^2 x \, dx$$

$$= \int (\tan^8 x + \tan^6 x) \sec^2 x \, dx$$

$$= \boxed{\frac{\tan^9 x}{9} + \frac{\tan^7 x}{7} + C}$$

II 3.10 Alex Sarver

$\int \sec^7 x \tan^5 x \, dx$

let $u = \sec x$

$du = \sec x \tan x \, dx$

$= \int \sec^6 x \tan^4 x \sec x \tan x \, dx$

$= \int \sec^6 x (\sec^2 x - 1)^2 \sec x \tan x \, dx$

$= \int \sec^6 x (\sec^4 x - 2\sec^2 x + 1) \sec x \tan x \, dx$

$= \int (\sec^{10} x - 2\sec^8 x + \sec^6 x) \sec x \tan x \, dx$

$= \int \sec^{10} x \sec x \tan x \, dx - 2\int \sec^8 x \sec x \tan x \, dx + \int \sec^6 x \sec x \tan x \, dx$

$\boxed{= \frac{1}{11}\sec^{11} x - \frac{2}{9}\sec^9 x + \frac{1}{7}\sec^7 x + C}$