

Arc length 5.1: $y = \ln(1-x^2)$, $x=0 \dots \frac{1}{2}$

$$\int_a^b \sqrt{1+f'(x)^2} dx \quad y' = \frac{-2x}{1-x^2}$$

$$\int_0^{1/2} \sqrt{1 + \frac{4x^2}{(1-x^2)^2}} dx \quad (y)'' = \frac{4x^2}{(1-x^2)^2}$$

$$\int_0^{1/2} \sqrt{\frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}} dx$$

$$\int_0^{1/2} \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}} dx \quad \frac{-1}{-x^2+1} \sqrt{x^2+1}$$

$$\int_0^{1/2} \frac{1+x^2}{1-x^2} dx \quad \frac{-(x^2-1)}{2}$$

$$\int_0^{1/2} -1 + \frac{2}{1-x^2} dx$$

$$\int_0^{1/2} -1 dx + \int_0^{1/2} \frac{2}{1-x^2} dx$$

$$2 \cdot \int \frac{1}{1-x^2} \Rightarrow \frac{a}{1+x} + \frac{b}{1-x}$$

$$2 \cdot \int \frac{1}{2} \left(\frac{1}{1+x} \right) + \frac{1}{2} \left(\frac{1}{1-x} \right) \frac{a-ax+b+bx}{1-x^2}$$

$$\int \frac{1}{1+x} + \frac{1}{1-x} dx \quad a+b=1$$

$$\ln(1+x) - \ln(1-x) \quad a-b=0$$

$$a=b=\frac{1}{2}$$

$$= \int_0^{1/2} -1 + \frac{2}{1-x^2} dx$$

$$= (-x + \ln(1+x) - \ln(1-x)) \Big|_0^{1/2}$$

$$= 0.5986$$

S.2 Arc Length Timmy

$$y = \cosh(x) \quad x = 0..1$$

$$y' = \sinh(x)$$

$$\int_0^1 \sqrt{1 + \sinh^2(x)} dx$$

$$\int_0^1 \cosh(x) dx$$

$$\sinh(x) \Big|_0^1$$

$$\sinh(1) - \sinh(0)$$

$$\boxed{\sinh(1) \text{ or } 1.17}$$

AL 5.3 $y = e^x - 1$

$$\int_{-1}^1 \sqrt{1 + y'(x)^2} dx$$

$$\int_0^1 \sqrt{1 + e^{2x}} dx$$

$$\int_0^1 \sqrt{1 + \tan^2 \theta} \cdot \frac{\sec^2 \theta}{\tan \theta} d\theta$$

$$\int_0^1 \sec \theta \cdot \frac{\sec^2 \theta}{\tan \theta} d\theta$$

$$\int \frac{1}{\cos \theta \cdot \sin \theta} d\theta$$

$$\int \frac{\sin^2 \theta \cdot \cos \theta}{\cos^2 \theta \cdot \sin \theta} d\theta$$

$$\int \frac{\sin^2 \theta}{\cos^2 \theta \cdot \sin \theta} d\theta \quad \int \frac{\cos^2 \theta}{\cos^2 \theta \cdot \sin \theta} d\theta$$

$$\begin{aligned} x &= e^x \\ y_1 &= e^x \\ y_2 &= 2x \\ y &= e \\ e^x &= \tan \theta \\ e^x dx &= \sec^2 \theta \\ dx &= \frac{\sec^2 \theta}{e^x} \end{aligned}$$

$$\begin{aligned} &\int \sec \theta \tan \theta d\theta + \int \sec \theta d\theta \\ &\ln |\csc \theta \cot \theta| + \sec \theta \Big|_0^1 \\ &\ln \left| \frac{\sqrt{1 + e^{2x}} - 1}{e^x} + \sqrt{1 + e^{2x}} \right|_0^1 \\ &\underline{\underline{2.0035}} \end{aligned}$$

Dane

Arc Length 5.4

$$x = \frac{t}{1+t}, y = \sqrt{1+t}, t = 0..2$$

$$= \int_0^2 \sqrt{\left(\frac{1}{(1+t)^2}\right)^2 + \left(\frac{1}{1+t}\right)^2} dt$$

$$= \int_0^2 \sqrt{\frac{1}{(1+t)^4} + \frac{1}{(1+t)^2}} dt$$

$$= \int_0^2 \sqrt{\frac{1+(1+t)^2}{(1+t)^4}} dt$$

$$u = t+1 \quad du = dt$$

$$= \int_1^3 \frac{\sqrt{1+u^2}}{u^2} du$$

$$\rightarrow = \ln(\sqrt{t^2+1}+t) - \frac{(t^2+1)^{3/2}}{t} + t\sqrt{t^2+1}$$

$$= \ln(\sqrt{10}+3) - \left(\frac{10^{3/2}}{3}\right) + 3\sqrt{10} \dots$$

$$\dots \ln(\sqrt{2}+1) - 2^{3/2} + \sqrt{2}$$

$$= 1.297$$

Korean

5.1 Mustafa

$$y = \sqrt{4-x^2}, x = -1 \dots 1 \text{ x axis}$$

$$f(x) = (4-x^2)^{1/2}$$

$$f'(x) = \frac{1}{2} (4-x^2)^{-1/2} \cdot -2x = -x(4-x^2)^{-1/2}$$

$$(f'(x))^2 = \frac{x^2}{4-x^2}$$

$$SA = \int_{-1}^1 2\pi \sqrt{4-x^2} \cdot \sqrt{1 + \frac{x^2}{4-x^2}} dx$$

$$= \int_{-1}^1 2\pi \cdot 2 \frac{1}{\sqrt{4-x^2}} \cdot \sqrt{4-x^2} dx$$

$$= 4\pi x \Big|_{-1}^1 = 8\pi$$

S.A. 5.2 Alex Jarver

$$x = \sqrt{y} \quad [1, 1] \dots [2, 4]$$

$$g'(y) = \frac{1}{2} y^{-1/2}$$

$$[g'(y)]^2 = \frac{1}{4} \cdot y^{-1}$$

$$2\pi \int_{-4}^4 y^{1/2} \sqrt{1 + \frac{1}{4y}} dy$$

$$2\pi \int_{-4}^4 \sqrt{y r y} dy$$

$$2\pi \int_{-4}^4 \frac{1}{2} \cdot \sqrt{4 r y} dy$$

$$\pi \int_{-4}^4 \sqrt{4y+1} dy$$

$$\frac{2}{3} \pi (4y+1)^{3/2} \Big|_{-4}^4$$
$$\frac{2\pi}{3} (17\sqrt{17} - 5\sqrt{5})$$

$$123.386$$

S.A. 5.3

Holla

$$y = e^x \quad x = 0 \dots 1 \quad x \text{ axis}$$

$$\int_0^1 2\pi e^x \sqrt{1 + (e^x)^2}$$

$$2\pi \int_0^1 \tan \theta \sqrt{\sec^2 \theta} \cdot \frac{\sec^2 \theta}{\tan \theta} d\theta$$

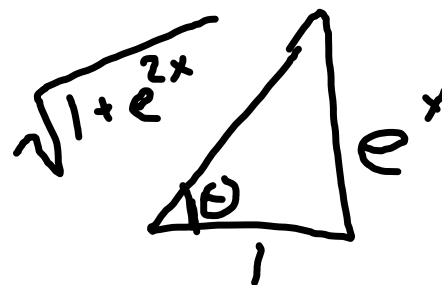
$$2\pi \int_0^1 \sec^3 \theta d\theta$$

$$e^x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$dx = \frac{\sec^2 \theta}{e^x} d\theta$$

$$2\pi \left(\frac{\sec \theta \tan \theta}{2} + \frac{\ln |\sec \theta + \tan \theta|}{2} \right) \Big|_0^1$$



$$2\pi \left(\frac{\sqrt{1 + e^{2x}} \cdot e^x}{2} + \frac{\ln |\sqrt{1 + e^{2x}} + e^x|}{2} \right) \Big|_0^1$$

$$= 22.943$$

S.A 5.4

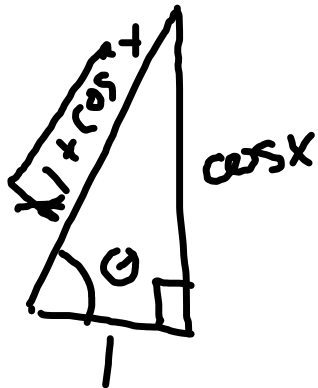
Amy

$y = \sin(x)$ $x=0 \dots \pi$
 x -axis

$$2\pi \int_0^\pi \sin x \sqrt{1 + \cos^2 x} dx$$

$\tan \theta = \cos x$

$\sec^2 \theta d\theta = -\sin x dx$



~~$$-2\pi \int_0^\pi \frac{\sin x}{\sin x} \sqrt{1 + \tan^2 \theta} \cdot \sec^2 \theta d\theta$$~~

$$-2\pi \int_0^\pi \sec^2 \theta \sec \theta d\theta$$

$$-2\pi \int_0^\pi (1 + \tan^2 \theta) \sec \theta d\theta$$

$$-2\pi \int_0^\pi \sec \theta d\theta - 2\pi \int_0^\pi \sec \theta \tan^2 \theta d\theta$$

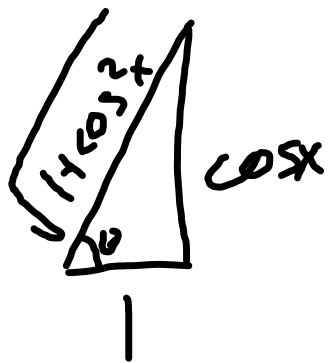
$u = \tan \theta$ $dv = \sec \theta \tan \theta d\theta$

$du = \sec^2 \theta d\theta$ $v = \sec \theta$

$$-2\pi \tan \theta \sec \theta + 2\pi \int \sec^3 \theta d\theta - 2\pi \ln |\sec \theta + \tan \theta| \Big|_0^\pi = 2\pi \int_0^\pi \sec^3 \theta d\theta$$

$$-2\pi \tan \theta \sec \theta - 2\pi \ln |\sec \theta + \tan \theta| \Big|_0^\pi = -4\pi \int_0^\pi \sec^3 \theta d\theta$$

$$-2\pi \tan \theta \sec \theta - 2\pi \ln |\sec \theta + \tan \theta| \Big|_0^\pi = -2\pi \int_0^\pi \sec^3 \theta d\theta$$



$$- \pi \cos x \sqrt{1 + \cos^2 x} \Big|_0^{\pi} - \pi \ln \left| \sqrt{1 + \cos^2 x} + \cos x \right| \Big|_0^{\pi}$$

14.4

5.5
Elizabeth

$$y = \cosh(x)$$

$$y' = \sinh(x)$$

$$2\pi \int_0^1 \cosh x \cdot \sqrt{1 + \sinh^2(x)} dx$$

$$2\pi \int_0^1 \cosh x \cdot \sqrt{\cosh^2 x} dx$$

$$2\pi \int_0^1 \cosh^2 x dx$$

$$2\pi \int_0^1 \left(\frac{e^x + e^{-x}}{2} \right)^2 dx$$

$$2\pi \int_0^1 \frac{e^{2x} + 2 + e^{-2x}}{4} dx$$

$$2\pi \left(\frac{e^{2x}}{8} + \frac{x}{2} + \frac{e^{-2x}}{8} \right) \Big|_0^1$$

$$\pi \left(\frac{e^2}{4} + 1 + \frac{-e^{-2}}{4} - \frac{1}{4} + \frac{1}{4} \right)$$

$$= \frac{\pi e^2}{4} - \frac{\pi e^{-2}}{4} + \pi$$

$$= 8.839$$

David Greenberg 5.8 S.A.

$$\int_0^1 2\pi t^3 \sqrt{9t^4 + 16t^6} dt$$

$$\frac{4}{3}t = \tan x$$

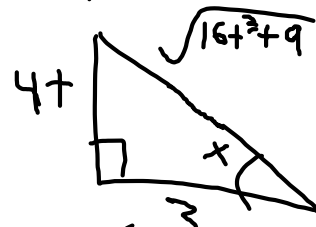
$$2\pi \int_0^1 t^3 \sqrt{9t^4 (1 + \frac{16}{9}t^2)} dt$$

$$t = \frac{3}{4} \tan x$$

$$dt = \frac{3}{4} \sec^2 x dx$$

$$6\pi \int_0^1 t^5 \sqrt{(\frac{4}{3}t)^2 + 1} dt$$

$$6\pi \int_0^1 (\frac{3}{4} \tan x)^5 \cdot \frac{3}{4} \sec^3 x dx$$



$$K \int_0^1 \tan^5 x \sec^3 x dx \quad K = 6 \cdot (\frac{3}{4})^6$$

$$K \int_0^1 \tan x (\sec^2 x - 1)^2 \sec x dx$$

$$K \int_0^1 (\sec^6 x - 2\sec^4 x + \sec^2 x) \sec x \tan x dx$$

$$K \left(\frac{u^7}{7} - \frac{2}{5} u^5 + \frac{u^3}{3} \right) \Big|_0^1 = 4.7$$

$$u = \sec x$$

$$\sec x = \frac{\sqrt{16t^2 + 9}}{3}$$

5.6

$$x = r \cos(t)$$

$$x'(t) = -r \sin(t)$$

$$(x'(t))^2 = r^2 \sin^2(t)$$

$$y = r \sin(t)$$

$$y'(t) = r \cos(t)$$

$$(y'(t))^2 = r^2 \cos^2(t)$$

$$t = 0 \dots \pi$$

David
Sarver

$$2\pi \int_0^\pi r \sin(t) \sqrt{r^2 \sin^2(t) + r^2 \cos^2(t)}$$

$$2\pi \int_0^\pi r^2 \sin(t)$$

$$2\pi r^2 \int_0^\pi \sin(t)$$

$$2\pi r^2 (-\cos(t)) \Big|_0^\pi$$

$$2\pi r^2 (1 + 1)$$

$$\boxed{4\pi r^2}$$

5.7 $r = \text{constant}, x = r\left(\frac{t}{2} - \sin\left(\frac{t}{2}\right)\right) \quad y = r(1 - \cos\left(\frac{t}{2}\right))$

Mat $x' = r - r \cos x$
 $2 \quad y' = r \sin x$

$$2\pi r \int_{\frac{0}{2\pi}}^{\frac{2\pi}{2\pi}} (1 - \cos t) \sqrt{1 + \cos^2 t - 2(\cos t + \sin^2 t) dt}$$

$$\sqrt{2} \cdot 2 \cdot \pi r \int_{\frac{0}{2\pi}}^{\frac{2\pi}{2\pi}} \sqrt{1 - \cos t} (1 - \cos t) dt$$

$t = 2\theta \rightarrow \sqrt{1 - 2\sin^2 \theta} \cdot (1 - 2\sin^2 \theta) dt$

$$2 \cdot 2 \cdot \sqrt{2} \cdot \sqrt{2} \cdot 2 \cdot \pi r \int_0^{\pi} \sqrt{\sin^2 \theta} \cdot \sin^2 \theta$$

$$16\pi r \int_0^{2\pi} \sin^2 \theta d\theta$$

$$\int_0^{2\pi} \sin x (1 - \cos^2 x) dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$\int (1 - u^2) du \rightarrow -u + \frac{u^3}{3} = \frac{\cos^3 x}{3} - \cos x \quad x = \frac{\pi}{2}$$

$$16\pi r \left(\frac{\cos^3 \frac{\pi}{2}}{3} - \cos \frac{\pi}{2} \right) \Big|_0^{2\pi} = 32\pi r - \frac{32\pi r}{3}$$

$$\frac{64\pi r}{3}$$