Arc length 5.1 : $y = \ln(1-x^2)$, $x = 0 .. \frac{1}{2}$

$$\int_a^b \sqrt{1+f'(x)^2}\,dx \qquad y' = \frac{-2x}{1-x^2}$$

$$\int_0^{1/2} \sqrt{1+\frac{4x^2}{(1-x^2)^2}}\,dx \qquad (y')^2 = \frac{4x^2}{(1-x^2)^2}$$

$$\int_0^{1/2} \sqrt{\frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}}\,dx$$

$$\int_0^{1/2} \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}}\,dx$$

$$-x^2+1 \overline{)\,x^2+1}$$
$$\frac{-1}{\quad}$$
$$\frac{-(x^2-1)}{2}$$

$$\int_0^{1/2} \frac{1+x^2}{1-x^2}\,dx$$

$$\int_0^{1/2} -1 + \frac{2}{1-x^2}\,dx$$

$$\int_0^{1/2} -1\,dx + \boxed{\int_0^{1/2} \frac{2}{1-x^2}\,dx}$$

$$2 \cdot \int \frac{1}{1-x^2} \Rightarrow \frac{a}{1+x} + \frac{b}{1-x}$$

$$2 \cdot \int \frac{1}{2}\left(\frac{1}{1+x}\right) + \frac{1}{2}\left(\frac{1}{1-x}\right) \qquad \frac{a-ax+b+bx}{1-x^2}$$

$$\int \frac{1}{1+x} + \frac{1}{1-x}\,dx \qquad a+b=1$$

$$\ln(1+x) - \ln(1-x) \qquad a-b=0$$

$$a=b=\frac{1}{2}$$

$$= \int_0^{1/2} -1 + \int_0^{1/2} \frac{2}{1-x^2}\,dx$$

$$= \left(-x + \ln(1+x) - \ln(1-x)\right)\Big|_0^{1/2}$$

$$\boxed{= 0.5986}$$

5.2 Arc Length Timmay

$y = \cosh(x) \quad x = 0..1$

$y'_x = \sinh x$

$$\int_0^1 \sqrt{1+\sinh^2 x}\,dx$$

$$\int_0^1 \cosh x\,dx$$

$$\sinh x \Big|_0^1$$

$$\sinh 1 - \sinh 0 \rightarrow 0$$

$$\boxed{\sinh 1 \quad \text{or} \quad 1.17}$$

AL 5.3 $y=e^x$ 0-1

$$\int_0^1 \sqrt{1+y'(x)^2}\,dx$$

$$\int_0^1 \sqrt{1+e^{2x}}\,dx$$

$$\int_0^1 \sqrt{1+\tan^2\theta}\cdot\frac{\sec^2\theta}{\tan\theta}\,d\theta$$

$$\int_0^1 \sec\theta\cdot\frac{\sec^2\theta}{\tan\theta}\,d\theta$$

$$\int \frac{1}{\cos^3\theta\cdot\sin\theta}\,d\theta$$

$$\int \frac{\sin^2\theta+\cos^2\theta}{\cos^3\theta\cdot\sin\theta}\,d\theta$$

$$\int \frac{\sin^2\theta}{\cos^3\theta\cdot\sin\theta}\,d\theta + \int \frac{\cos^2\theta}{\cos^3\theta\cdot\sin\theta}\,d\theta$$

$y=e^x$

$y'=e^x$

$y'^2=e^{2x}$

$e^x=\tan\theta$

$e^x\,dx=\sec^2\theta$

$dx=\frac{\sec^2\theta}{e^x}$

$$\int \sec\theta\tan\theta\,d\theta + \int \csc\theta\,d\theta$$

$$\ln|\csc\theta-\cot\theta| + \sec\theta\Big|_0^1$$

$$\ln\left|\frac{\sqrt{1+e^{2x}}-1}{e^x}\right| + \sqrt{1+e^{2x}}\Big|_0^1$$

2.0035

Dane

Arc Length 5.4

$$x = \frac{t}{1+t},\ y = \ln(1+t),\ t = 0..2$$

$$= \int_0^2 \sqrt{\left(\frac{1}{(x+1)^2}\right)^2 + \left(\frac{1}{1+t}\right)^2}\, dt$$

$$= \int_0^2 \sqrt{\frac{1}{(x+1)^4} + \frac{1}{(1+t)^2}}\, dt$$

$$= \int_0^2 \sqrt{\frac{1+(t+1)^2}{(t+1)^4}}\, dt$$

$u = t+1 \quad du = dt$

$$= \int_1^3 \frac{\sqrt{1+u^2}}{u^2}\, du$$

$$= \ln\left(\sqrt{t^2+1}+t\right) - \frac{(t^2+1)^{3/2}}{t} + t\sqrt{t^2+1}\Big|_1^3$$

$$= \ln\left(\sqrt{10}+3\right) - \left(\frac{10^{3/2}}{3}\right) + 3\sqrt{10} \ldots$$

$$\ldots \ln\left(\sqrt{2}+1\right) - 2^{3/2} + \sqrt{2}$$

$$\boxed{= 1.297}$$

Korean

5.1 Mustafa

$y = \sqrt{4 - x^2}, x = -1 \ldots 1$ x axis

$f(x) = (4 - x^2)^{1/2}$

$f'(x) = \frac{1}{2}(4 - x^2)^{-1/2} \cdot -2x = -x(4 - x^2)^{-1/2}$

$(f'(x))^2 = \frac{x^2}{4 - x^2}$

$$SA = \int_{-1}^{1} 2\pi \sqrt{4 - x^2} \cdot \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot 2 \frac{1}{\sqrt{4 - x^2}} \cdot \sqrt{4 - x^2}\, dx$$

$$= 4\pi x \Big|_{-1}^{1} = 8\pi$$

S.A. 5.2 Alex Jarver

$x=\sqrt{y}$ [1,1]. . . [2,4]

$g'(y)=\frac{1}{2}y^{-1/2}$

$[g'(y)]^2=\frac{1}{4}\cdot y^{-1}$

$2\pi\int_1^4 y^{1/2}\sqrt{1+\frac{1}{4y}}\,dy$

$2\pi\int_1^4 \sqrt{y+\frac{1}{4}}\,dy$

$2\pi\int_1^4 \frac{1}{2}\cdot\sqrt{4}\sqrt{y+\frac{1}{4}}\,dy$

$\pi\int_1^4 \sqrt{4y+1}\,dy$

$\frac{2}{3}\pi(4y+1)^{3/2}\Big|_1^4$

$\frac{2\pi}{3}(17\sqrt{17}-5\sqrt{5})$

123.386

S.A. 5.3

HollA

$e^x = \tan\theta$

$dx\, e^x = \sec^2\theta\, d\theta$

$dx = \frac{\sec^2\theta}{e^x} d\theta$

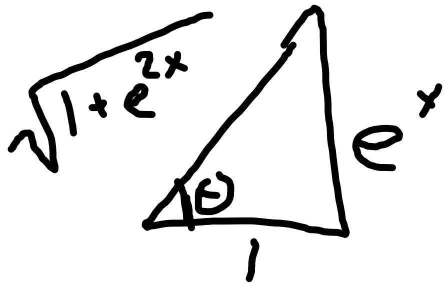
$y = e^x \quad x = 0..1 \quad x\text{ axis}$

$$\int_0^1 2\pi e^x \sqrt{1+(e^x)^2}$$

$$2\pi \int_0^1 \tan\theta \sqrt{\sec^2\theta} \cdot \frac{\sec^2\theta}{\tan\theta}\, d\theta$$

$$2\pi \int_0^1 \sec^3\theta\, d\theta$$

$$2\pi\left(\frac{\sec\theta\tan\theta}{2} + \frac{\ln|\sec\theta + \tan\theta|}{2}\right)\Bigg|_0^1$$



$$2\pi\left(\frac{\sqrt{1+e^{2x}} \cdot e^x}{2} + \frac{\ln\left|\sqrt{1+e^{2x}} + e^x\right|}{2}\right)\Bigg|_0^1$$

$$\boxed{= 22.943}$$

S.A 5.4

Amy

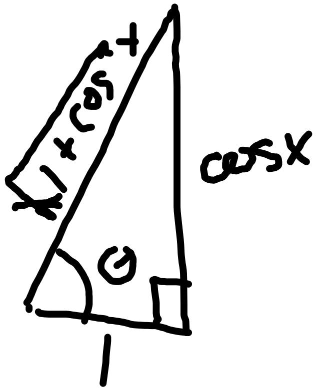
$y = \sin(x)$ $x = 0..\pi$

x-axis

$$2\pi \int_0^{\pi} \sin x \sqrt{1+\cos^2 x}\, dx$$

$\tan\theta = \cos x$

$\sec^2\theta\, d\theta = -\sin x\, dx$



$$-2\pi \int_0^{\pi} \frac{\cancel{\sin x}}{\cancel{\sin x}} \cancel{\sqrt{1+\tan^2\theta}}^{\sec\theta} \cdot \sec^2\theta\, d\theta$$

$$-2\pi \int_0^{\pi} \sec^2\theta \sec\theta\, d\theta$$

$$-2\pi \int_0^{\pi} (1+\tan^2\theta) \sec\theta\, d\theta$$

$$-2\pi \int_0^{\pi} \sec\theta\, d\theta - 2\pi \int \sec\theta \tan^2\theta\, d\theta$$

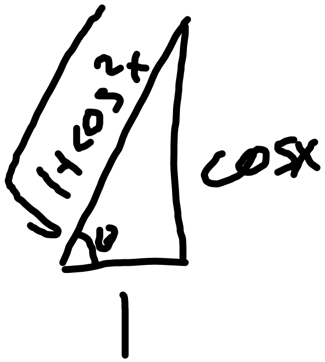
$u = \tan\theta$ $dv = \sec\theta\tan\theta\, d\theta$

$du = \sec^2\theta\, d\theta$ $v = \sec\theta$

$$-2\pi \tan\theta\sec\theta + 2\pi \int_0^{\pi} \sec^3\theta\, d\theta - 2\pi \ln|\sec\theta \pm \tan\theta| \Big|_0^{\pi} = 2\pi \int_0^{\pi} \sec^3\theta\, d\theta$$

$$-2\pi \tan\theta\sec\theta - 2\pi \ln|\sec\theta + \tan\theta| \Big|_0^{\pi} = -4\pi \int_0^{\pi} \sec^3\theta\, d\theta$$

$$-\pi \tan\theta\sec\theta - \pi \ln|\sec\theta + \tan\theta| \Big|_0^{\pi} = -2\pi \int_0^{\pi} \sec^3\theta\, d\theta$$



$$-\pi\cos x\sqrt{1+\cos^2 x}\,\Big|_0^{\pi} - \pi\ln\left|\sqrt{1+\cos^2 x}+\cos x\right|\,\Big|_0^{\pi}$$

$$\boxed{14.4}$$

5.5

Elizabeth

$y = \cosh(x)$

$y' = \sinh(x)$

$$2\pi \int_0^1 \cosh x \cdot \sqrt{1+\sinh^2(x)}\, dx$$

$$2\pi \int_0^1 \cosh x \cdot \sqrt{\cosh^2 x}\, dx$$

$$2\pi \int_0^1 \cosh^2 x\, dx$$

$$2\pi \int_0^1 \left(\frac{e^x + e^{-x}}{2}\right)^2 dx$$

$$2\pi \int_0^1 \frac{e^{2x} + 2 + e^{-2x}}{4}\, dx$$

$$2\pi \left(\frac{e^{2x}}{8} + \frac{x}{2} + \frac{e^{-2x}}{8}\right)\Bigg|_0^1$$

$$\pi\left(\frac{e^2}{4} + 1 + \frac{-e^{-2}}{4} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}\right)$$

$$= \frac{\pi e^2}{4} - \frac{\pi e^{-2}}{4} + \pi$$

$$\boxed{= 8.839}$$

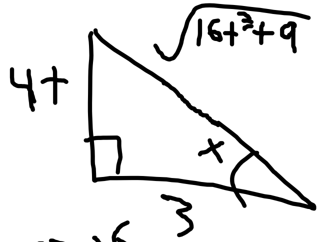
David Greenberg 5.8 S.A.

$$\int_0^1 2\pi t^3 \sqrt{9t^4+16t^6}\,dt \qquad \frac{4}{3}t = \tan x$$

$$2\pi\int_0^1 t^3\sqrt{9t^4\left(1+\frac{16}{9}t^2\right)}\,dt \qquad t = \tfrac{3}{4}\tan x$$

$$6\pi\int_0^1 t^5\sqrt{\left(\tfrac{4}{3}t\right)^2+1}\,dt \qquad dt = \frac{3}{4}\sec^2 x\,dx$$

$$6\pi\int_0^1 \left(\tfrac{3}{4}\tan x\right)^5\cdot \tfrac{3}{4}\cdot\sec^3 x\,dx$$



$$K\int_0^1 \tan^5 x\sec^3 x\,dx \qquad K = 6\cdot\left(\tfrac{3}{4}\right)^6$$

$$K\int_0^1 \tan x\left(\sec^2 x-1\right)^2\sec x\,dx$$

$$K\int_0^1 \left(\sec^6 x-2\sec^4 x+\sec^2 x\right)\sec x\tan x\,dx$$

$$u=\sec x$$

$$K\left(\frac{u^7}{7}-\frac{2}{5}u^5+\frac{u^3}{3}\right)\Big|_0^1 = 4.7$$

$$\sec x = \frac{\sqrt{16t^2+9}}{3}$$

5.6 $x = r\cos(t)$ $y = r\sin(t)$ $t = 0 \ldots \pi$

David Sarver

$x'(t) = r\sin(t)$ $y'(t) = r\cos(t)$

$(x'(t))^2 = r^2\sin^2(t)$ $(y'(t))^2 = r^2\cos^2(t)$

$$2\pi \int_0^\pi r\sin(t)\sqrt{r^2\sin^2(t) + r^2\cos^2(t)}$$

$$2\pi \int_0^\pi r^2 \sin(t)$$

$$2\pi r^2 \int_0^\pi \sin(t)$$

$$2\pi r^2 \left(-\cos(t)\right)\Big|_0^\pi$$

$$2\pi r^2 (1+1)$$

$$\boxed{4\pi r^2}$$

5.7 $r = \text{constant},\ x = r(t - \sin(t))$ $\quad y = r(1-\cos(t))$

Mat 2

$x' = r - r\cos x$

$y' = r\sin x$

$$2\pi r\int_0^{2\pi}(1-\cos t)\sqrt{1+\cos^2 t - 2\cos t + \sin^2 t}\,dt$$

$t = 2\theta$

$$\sqrt{2}\cdot 2\cdot\pi r\int_0^{2\pi}\sqrt{1-\cos t}\,(1-\cos t)$$

$\sqrt{1 - 2\sin^2\theta - 1}$

$(1 - 2\sin^2\theta - 1)$

$$2\cdot 2\cdot\sqrt{2}\cdot\sqrt{2}\cdot 2\cdot 2\pi r\int_0^{2\pi}\sqrt{\sin^2\theta}\cdot\sin^2\theta$$

$$16\pi r\int_0^{2\pi}\sin^3\theta\, d\theta$$

$$\int_0^{2\pi}\sin x(1-\cos^2 x)\,dx$$

$u=\cos x$

$du=-\sin x\,dx$

$x=\frac{\pi}{2}$

$$-\int(1-u^2)\,du \to -u+\frac{u^3}{3}=\frac{\cos^3 x}{3}-\cos x$$

$$16\pi r\left(\frac{\cos^3\frac{\pi}{2}}{3}-\cos\frac{\pi}{2}\right)\Bigg|_0^{2\pi}=32\pi r-\frac{32\pi r}{3}$$

$$\frac{64\pi r}{3}$$