**Formulas for Arc (Curve) Length in the Plane:**

$$\textit{Arc Length for a curve } y = \mathrm{f}(x): \quad \int_a^b \sqrt{1 + \mathrm{f}\ '(x)^2}\, dx$$

$$\textit{Arc Length for a curve } x = \mathrm{g}(y): \quad \int_c^d \sqrt{1 + \mathrm{g}\ '(y)^2}\, dy$$

$$\textit{Arc Length for a parametric curve } [x, y] = [\mathrm{x}(t), \mathrm{y}(t)]: \quad \int_{t_1}^{t_2} \sqrt{\mathrm{x}\ '(t)^2 + \mathrm{y}'(t)^2}\, dt$$

**Arc Length Exercises:**

*Arc Length Exercise 5.1 [Use Integration by Parts]*: $y = \ln(1 - x^2),\ x = 0..\frac{1}{2}$

*Arc Length Exercise 5.2 [Use Hyperbolic Functions]*: $y = cosh(x) = \dfrac{e^x + e^{(-x)}}{2},\ x = 0..1$

*Arc Length Exercise 5.3*: $y = e^x,\ x = 0..1$

*Arc Length Exercise 5.4 :* $x = \dfrac{t}{1+t}, \quad y = \ln(1+t),\ t = 0..2$

**Formulas for Surface Area:**

$$Surface\ Area\ rotate\ about\ x-axis = \int_a^b 2\,\pi\,\mathrm{f}(x)\,\sqrt{1+\mathrm{f}\,'(x)^2}\,dx$$

$$Surface\ Area\ rotate\ about\ y-axis = \int_c^d 2\,\pi\,\mathrm{g}(y)\,\sqrt{1+\mathrm{g}\,'(y)^2}\,dy$$

$$Parametric\ Surface\ Area\ rotate\ about\ x-axis = \int_{t_1}^{t_2} 2\,\pi\,\mathrm{y}(t)\,\sqrt{\mathrm{x}\,'(t)^2+\ \mathrm{y}'(t)^2}\,dt$$

$$Parametric\ Surface\ Area\ rotate\ about\ y-axis = \int_{t_1}^{t_2} 2\,\pi\,\mathrm{x}(t)\,\sqrt{\mathrm{x}\,'(t)^2+\ \mathrm{y}'(t)^2}\,dt$$

**Surface Area Exercises:**

*Surface Area Exercise 5.1 :* $y = \sqrt{4-x^2},\ x = -1..1,$ *rotate about* $x-$ *axis.*

*Surface Area Exercise 5.2 :* $x = \sqrt{y},$ *from* $[1, 1]$ *to* $[2, 4]$*, choose axis.*

*Surface Area Exercise 5.3 :* $y = e^x$, $x = 0..1$, *rotate about* $x - axis$.

*Surface Area Exercise 5.4 :* $y = \sin(x)$, $x = 0..\pi$, *rotate about* $x - axis$.

*Surface Area Exercise 5.5 :* $y \; = \; cosh(x) = \dfrac{e^x + e^{(-x)}}{2}$, $x = 0..1$, *rotate about* $x - axis$.

*Surface Area Exercise 5.6 :* $r$ *is constant,* $x = r\cos(t)$, $y = r\sin(t)$, $t = 0..\pi$, *rotate about* $x - axis$.

*Surface Area Exercise 5.7 :* $r$ *is constant,* $x = r\,(t - \sin(t))$, $y = r\,(1 - \cos(t))$, $t = 0..2\,\pi$, *rotate about* $x - axis$.

*Surface Area Exercise 5.8 :* $x = t^3$, $y = t^4$, $t = 0..1$, *rotate about* $y - axis$.