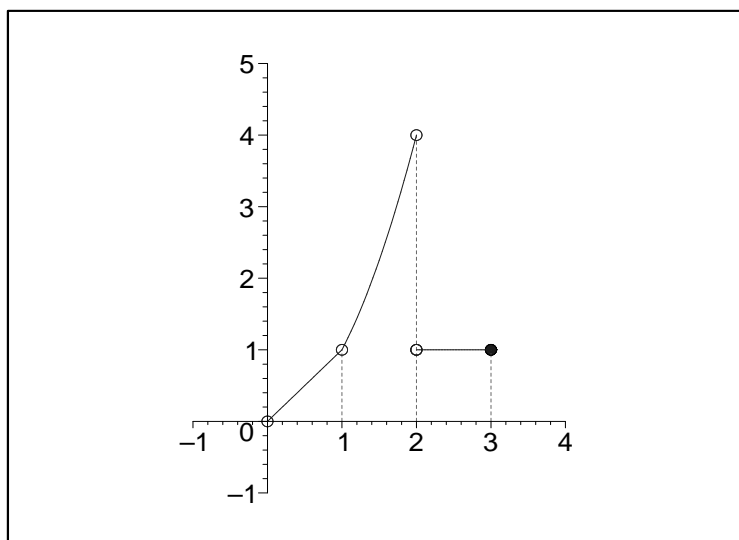


Calculus Exploration 1: Limits of Piecewise Functions

Limits of Piecewise Functions: Example 1



$$f(x) = \begin{cases} x & 0 < x \text{ and } x < 1 \\ x^2 & 1 < x \text{ and } x < 2 \\ 1 & 2 < x \text{ and } x \leq 3 \\ \text{undefined} & \text{otherwise} \end{cases}$$

Limiting value that $f(x)$ approaches as x approaches 0 from the left :

$$\lim_{x \rightarrow 0^-} f(x) = \text{undefined}, f(0) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 0 from the right :

$$\lim_{x \rightarrow 0^+} f(x) = 0, f(0) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 0 from either side :

$$\lim_{x \rightarrow 0} f(x) = \text{undefined}, f(0) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 1 from the left :

$$\lim_{x \rightarrow 1^-} f(x) = 1, f(1) = \text{undefined}$$

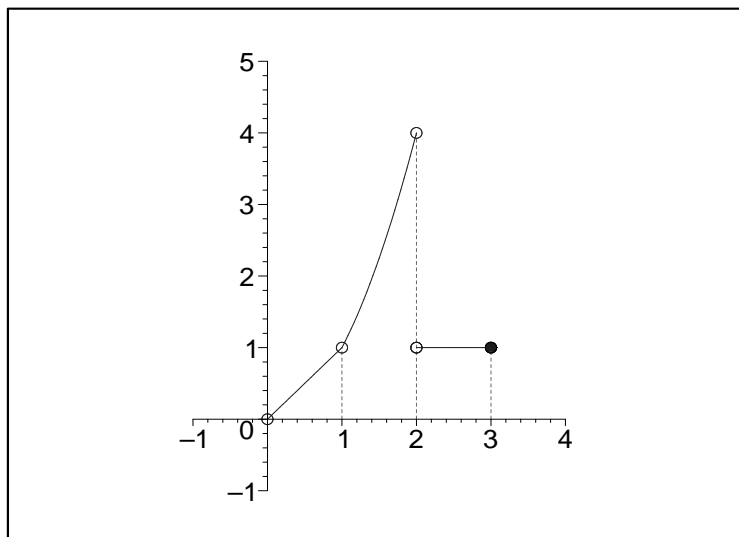
Limiting value that $f(x)$ approaches as x approaches 1 from the right :

$$\lim_{x \rightarrow 1^+} f(x) = 1, f(1) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 1 from either side :

$$\lim_{x \rightarrow 1} f(x) = 1, f(1) = \text{undefined}$$

Limits of Piecewise Functions: Example 1 (continued)



$$f(x) = \begin{cases} x & 0 < x \text{ and } x < 1 \\ x^2 & 1 < x \text{ and } x < 2 \\ 1 & 2 < x \text{ and } x \leq 3 \\ \text{undefined} & \text{otherwise} \end{cases}$$

Limiting value that $f(x)$ approaches as x approaches 2 from the left :

$$\lim_{x \rightarrow 2^-} f(x) = 4, f(2) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 2 from the right :

$$\lim_{x \rightarrow 2^+} f(x) = 1, f(2) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 2 from either side :

$$\lim_{x \rightarrow 2} f(x) = \text{undefined}, f(2) = \text{undefined}$$

Limiting value that $f(x)$ approaches as x approaches 3 from the left :

$$\lim_{x \rightarrow 3^-} f(x) = 1, f(3) = 1$$

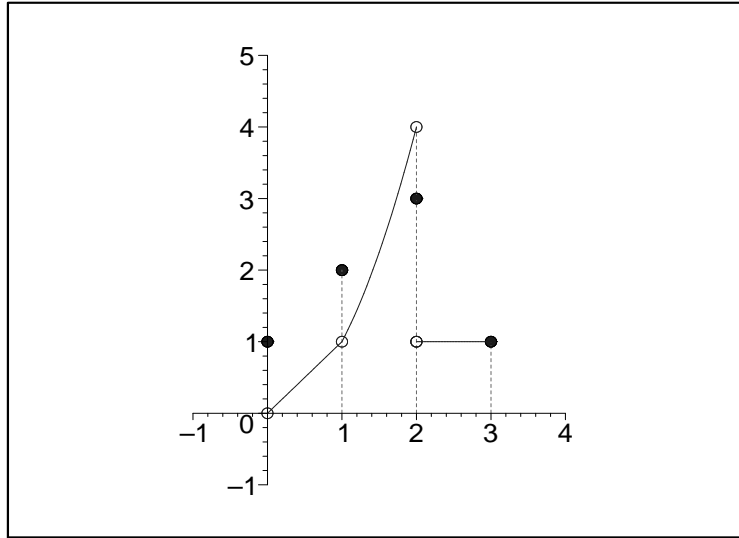
Limiting value that $f(x)$ approaches as x approaches 3 from the right :

$$\lim_{x \rightarrow 3^+} f(x) = \text{undefined}, f(3) = 1$$

Limiting value that $f(x)$ approaches as x approaches 3 from either side :

$$\lim_{x \rightarrow 3} f(x) = \text{undefined}, f(3) = 1$$

Limits of Piecewise Functions: Example 2



$$f(x) = \begin{cases} 1 & x = 0 \\ x & 0 < x \text{ and } x < 1 \\ 2 & x = 1 \\ x^2 & 1 < x \text{ and } x < 2 \\ 3 & x = 2 \\ 1 & 2 < x \text{ and } x \leq 3 \\ \text{undefined} & \text{otherwise} \end{cases}$$

Limiting value that $f(x)$ approaches as x approaches 0 from the left :

$$\lim_{x \rightarrow 0^-} f(x) = \text{undefined}, f(0) = 1$$

Limiting value that $f(x)$ approaches as x approaches 0 from the right :

$$\lim_{x \rightarrow 0^+} f(x) = 0, f(0) = 1$$

Limiting value that $f(x)$ approaches as x approaches 0 from either side :

$$\lim_{x \rightarrow 0} f(x) = \text{undefined}, f(0) = 1$$

Limiting value that $f(x)$ approaches as x approaches 1 from the left :

$$\lim_{x \rightarrow 1^-} f(x) = 1, f(1) = 2$$

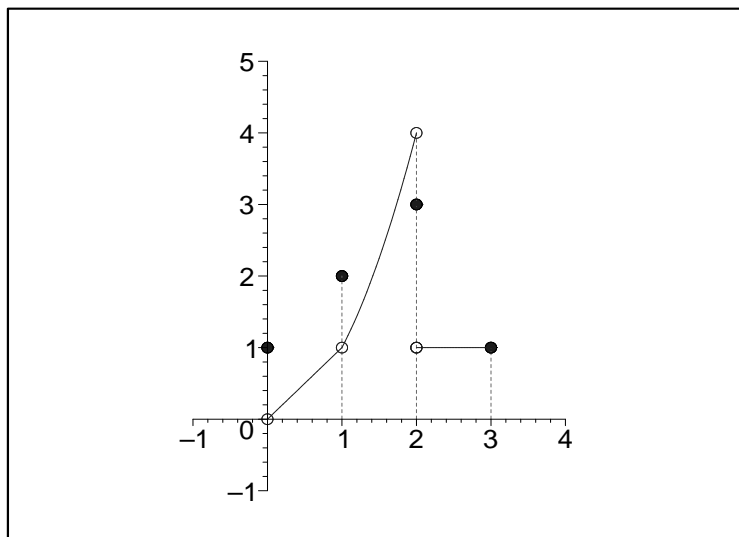
Limiting value that $f(x)$ approaches as x approaches 1 from the right :

$$\lim_{x \rightarrow 1^+} f(x) = 1, f(1) = 2$$

Limiting value that $f(x)$ approaches as x approaches 1 from either side :

$$\lim_{x \rightarrow 1} f(x) = 1, f(1) = 2$$

Limits of Piecewise Functions: Example 2 (continued)



$$f(x) = \begin{cases} 1 & x = 0 \\ x & 0 < x \text{ and } x < 1 \\ 2 & x = 1 \\ x^2 & 1 < x \text{ and } x < 2 \\ 3 & x = 2 \\ 1 & 2 < x \text{ and } x \leq 3 \\ \text{undefined} & \text{otherwise} \end{cases}$$

Limiting value that $f(x)$ approaches as x approaches 2 from the left :

$$\lim_{x \rightarrow 2^-} f(x) = 4, f(2) = 3$$

Limiting value that $f(x)$ approaches as x approaches 2 from the right :

$$\lim_{x \rightarrow 2^+} f(x) = 1, f(2) = 3$$

Limiting value that $f(x)$ approaches as x approaches 2 from either side :

$$\lim_{x \rightarrow 2} f(x) = \text{undefined}, f(2) = 3$$

Limiting value that $f(x)$ approaches as x approaches 3 from the left :

$$\lim_{x \rightarrow 3^-} f(x) = 1, f(3) = 1$$

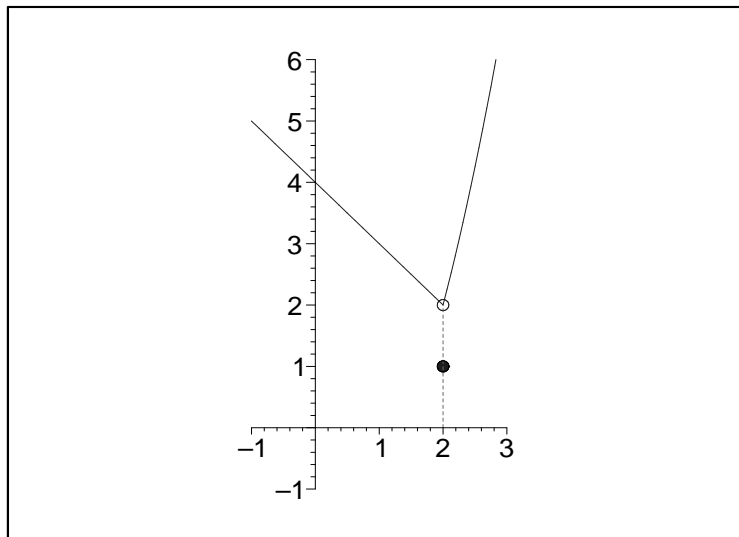
Limiting value that $f(x)$ approaches as x approaches 3 from the right :

$$\lim_{x \rightarrow 3^+} f(x) = \text{undefined}, f(3) = 1$$

Limiting value that $f(x)$ approaches as x approaches 3 from either side :

$$\lim_{x \rightarrow 3} f(x) = \text{undefined}, f(3) = 1$$

Limits of Piecewise Functions: Exercise 1



$$f(x) = \begin{cases} 4 - x & x < 2 \\ 1 & x = 2 \\ x^2 - 2 & 2 < x \end{cases}$$

Compute the following limits and fill in the blanks :

$$\lim_{x \rightarrow 0^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0} f(x) = \underline{\quad}, \quad f(0) = \underline{\quad}$$

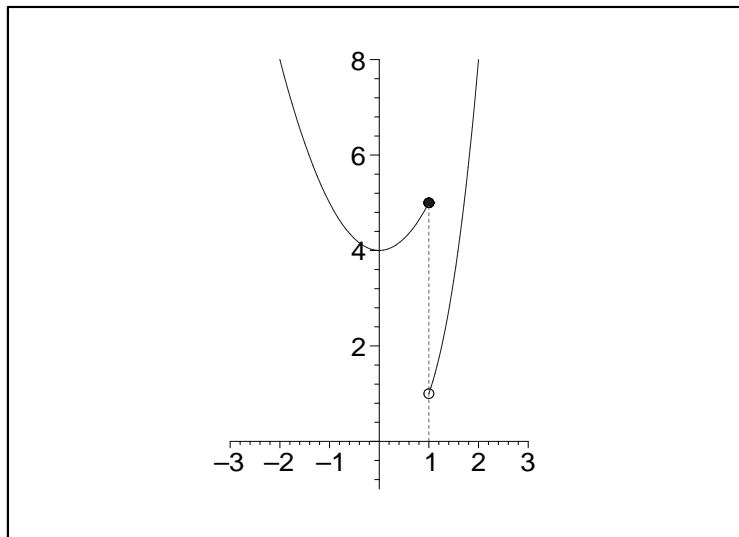
$$\lim_{x \rightarrow 1^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1} f(x) = \underline{\quad}, \quad f(1) = \underline{\quad}$$

$$\lim_{x \rightarrow 2^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2} f(x) = \underline{\quad}, \quad f(2) = \underline{\quad}$$

Redefine the function f so that it is continuous :

$$f(x) = \begin{cases} & x < 2 \\ & x = 2 \\ & 2 < x \end{cases}$$

Limits of Piecewise Functions: Exercise 2



$$f(x) = \begin{cases} x^2 + 4 & x < 1 \\ 5 & x = 1 \\ x^3 & 1 < x \end{cases}$$

Compute the following limits and fill in the blanks :

$$\lim_{x \rightarrow (-1)^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-1)^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-1)} f(x) = \underline{\quad}, \quad f(-1) = \underline{\quad}$$

$$\lim_{x \rightarrow 0^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0} f(x) = \underline{\quad}, \quad f(0) = \underline{\quad}$$

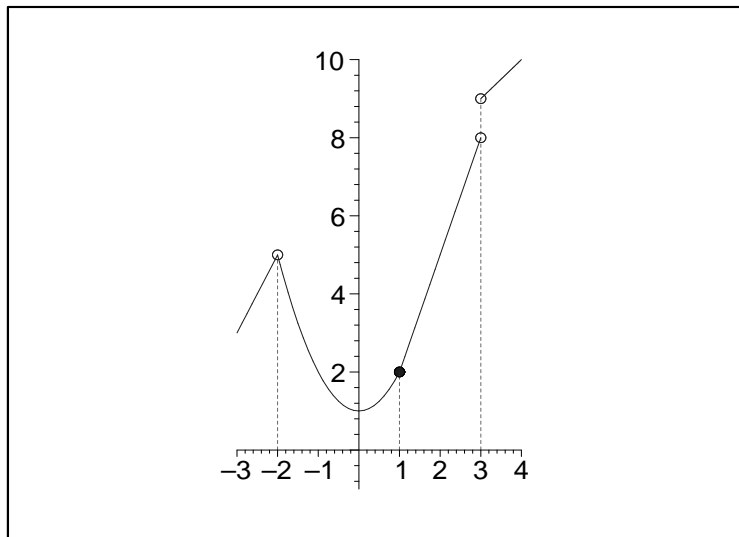
$$\lim_{x \rightarrow 1^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1} f(x) = \underline{\quad}, \quad f(1) = \underline{\quad}$$

$$\lim_{x \rightarrow 2^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2} f(x) = \underline{\quad}, \quad f(2) = \underline{\quad}$$

Redefine the function f so that it is continuous :

$$f(x) = \begin{cases} & x < 1 \\ & x = 1 \\ & 1 < x \end{cases}$$

Limits of Piecewise Functions: Exercise 3



$$f(x) = \begin{cases} 2x + 9 & x < -2 \\ x^2 + 1 & -2 < x \text{ and } x \leq 1 \\ 3x - 1 & 1 < x \text{ and } x < 3 \\ x + 6 & 3 < x \\ \text{undefined} & \text{otherwise} \end{cases}$$

Compute the following limits and fill in the blanks :

$$\lim_{x \rightarrow (-2)^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-2)^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-2)} f(x) = \underline{\quad}, \quad f(-2) = \underline{\quad}$$

$$\lim_{x \rightarrow (-1)^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-1)^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow (-1)} f(x) = \underline{\quad}, \quad f(-1) = \underline{\quad}$$

$$\lim_{x \rightarrow 0^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 0} f(x) = \underline{\quad}, \quad f(0) = \underline{\quad}$$

$$\lim_{x \rightarrow 1^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 1} f(x) = \underline{\quad}, \quad f(1) = \underline{\quad}$$

$$\lim_{x \rightarrow 2^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 2} f(x) = \underline{\quad}, \quad f(2) = \underline{\quad}$$

$$\lim_{x \rightarrow 3^-} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 3^+} f(x) = \underline{\quad}, \quad \lim_{x \rightarrow 3} f(x) = \underline{\quad}, \quad f(3) = \underline{\quad}$$

Redefine the function f so that it is continuous :

$$f(x) = \begin{cases} x < -2 \\ x = -2 \\ -2 < x \text{ and } x \leq 1 \\ 1 < x \text{ and } x < 3 \\ x = 3 \\ 3 < x \end{cases}$$

Limits of Piecewise Functions: Exercise 4

Read through three different calculus books and find a definition for each one of the following concepts that makes the most sense to you. When you find the one you like the best for each concept, write it in the appropriate place provided below.

Limit Definition 1.

$$\textit{Left hand limit, } \lim_{x \rightarrow a^-} f(x) = b$$

Limit Definition 2.

$$\textit{Right hand limit, } \lim_{x \rightarrow a^+} f(x) = b$$

Limit Definition 3.

$$\textit{Two - sided limit, } \lim_{x \rightarrow a} f(x) = b$$

Limit Definition 4.

$$\textit{The function } f(x) \textit{ is continuous at } x = a$$

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