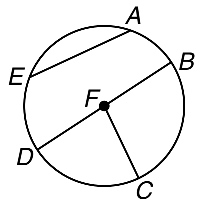
NAME ____ DATE ____________ PERIOD _____

10-1 Study Guide and Intervention

Circles and Circumference

Parts of Circles A **circle** consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.

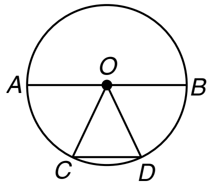


chord: $\overline{AE}$, $\overline{BD}$
radius: $\overline{FB}$, $\overline{FC}$, $\overline{FD}$
diameter: $\overline{BD}$

A segment or line can intersect a circle in several ways.

- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.

Example



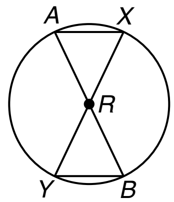
a. Name the circle.
The name of the circle is $\odot O$.

b. Name radii of the circle.
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$ are radii.

c. Name chords of the circle.
$\overline{AB}$ and $\overline{CD}$ are chords.

d. Name a diameter of the circle.
$\overline{AB}$ is a diameter.

Exercises



1. Name the circle.

2. Name radii of the circle.

3. Name chords of the circle.

4. Name diameters of the circle.

5. Find AR if AB is 18 millimeters.

6. Find AR and AB if RY is 10 inches.

7. Is $\overline{AB} \cong \overline{XY}$? Explain.

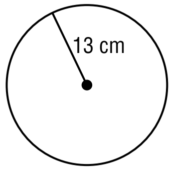
10-1 Study Guide and Intervention *(continued)*

Circles and Circumference

Circumference The **circumference** of a circle is the distance around the circle.

Circumference	For a circumference of C units and a diameter of d units or a radius of r units, $C = \pi d$ or $C = 2\pi r$.

Example **Find the circumference of the circle to the nearest hundredth.**



$C = 2\pi r$ Circumference formula

$= 2\pi(13)$ $r = 13$

≈ 81.68 Use a calculator.

The circumference is about 81.68 centimeters.

Exercises

Find the circumference of a circle with the given radius or diameter. Round to the nearest hundredth.

1. $r = 8$ cm

2. $r = 3\sqrt{2}$ ft

3. $r = 4.1$ cm

4. $d = 10$ in.

5. $d = \frac{1}{3}$ m

6. $d = 18$ yd

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

7. $r = 4$ cm

$d =$ ____________, $C =$ ____________

8. $d = 6$ ft

$r =$ ____________, $C =$ ____________

9. $r = 12$ cm

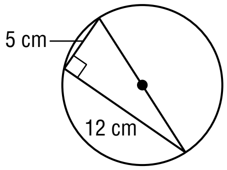
$d =$ ____________, $C =$ ____________

10. $d = 15$ in.

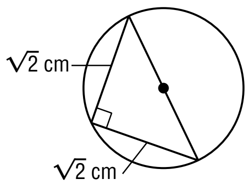
$r =$ ____________, $C =$ ____________

Find the exact circumference of each circle.

11.



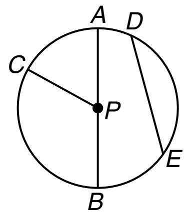
12.



10-1 Skills Practice

Circles and Circumference

For Exercises 1–5, refer to the circle.



1. Name the circle.

2. Name a radius.

3. Name a chord.

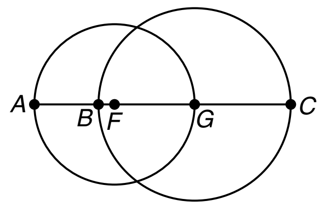
4. Name a diameter.

5. Name a radius not drawn as part of a diameter.

6. Suppose the diameter of the circle is 16 centimeters. Find the radius.

7. If $PC = 11$ inches, find AB.

The diameters of $\odot F$ and $\odot G$ are 5 and 6 units, respectively. Find each measure.



8. BF

9. AB

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

10. $r = 8$ cm

$d =$ ____________, $C \approx$ ____________

11. $r = 13$ ft

$d =$ ____________, $C \approx$ ____________

12. $d = 9$ m

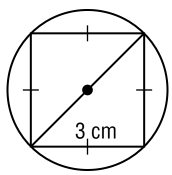
$r =$ ____________, $C \approx$ ____________

13. $C = 35.7$ in.

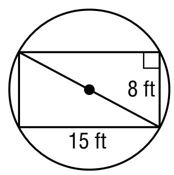
$d \approx$ ____________, $r \approx$ ____________

Find the exact circumference of each circle.

14.



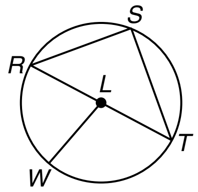
15.



10-1 Practice

Circles and Circumference

For Exercises 1–5, refer to the circle.



1. Name the circle.

2. Name a radius.

3. Name a chord.

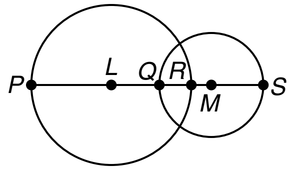
4. Name a diameter.

5. Name a radius not drawn as part of a diameter.

6. Suppose the radius of the circle is 3.5 yards. Find the diameter.

7. If $RT = 19$ meters, find LW.

The diameters of $\odot L$ and $\odot M$ are 20 and 13 units, respectively. Find each measure if $QR = 4$.



8. LQ

9. RM

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

10. $r = 7.5$ mm

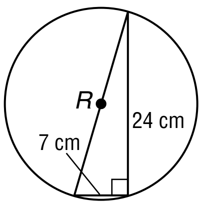
$d =$ ____________, $C \approx$ ____________

11. $C = 227.6$ yd

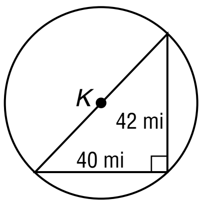
$d \approx$ ____________, $r \approx$ ____________

Find the exact circumference of each circle.

12.



13.



SUNDIALS For Exercises 14 and 15, use the following information.

Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

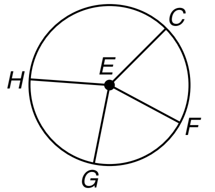
14. Find the radius of the sundial.

15. Find the circumference of the sundial to the nearest hundredth.

10-2 Study Guide and Intervention

Angles and Arcs

Angles and Arcs A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.

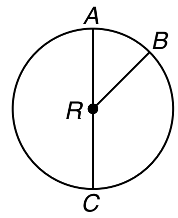


$\overset{\frown}{GF}$ is a minor arc.
$\overset{\frown}{CHG}$ is a major arc.
$\angle GEF$ is a central angle.

Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360. — $m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$
- The measure of a minor arc equals the measure of its central angle. — $m\overset{\frown}{CF} = m\angle CEF$
- The measure of a major arc is 360 minus the measure of the minor arc. — $m\overset{\frown}{CGF} = 360 - m\overset{\frown}{CF}$
- Two arcs are congruent if and only if their corresponding central angles are congruent. — $\overset{\frown}{CF} \cong \overset{\frown}{FG}$ if and only if $\angle CEF \cong \angle FEG$.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. **(Arc Addition Postulate)** — $m\overset{\frown}{CF} + m\overset{\frown}{FG} = m\overset{\frown}{CG}$

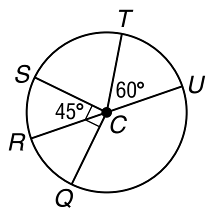
Example **In $\odot R$, $m\angle ARB = 42$ and $\overline{AC}$ is a diameter. Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{ACB}$.**



$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\overset{\frown}{AB} = 42$.
Thus $m\overset{\frown}{ACB} = 360 - 42$ or 318.

Exercises

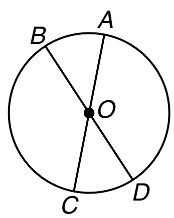
Find each measure.



1. $m\angle SCT$ 2. $m\angle SCU$

3. $m\angle SCQ$ 4. $m\angle QCT$

If $m\angle BOA = 44$, find each measure.



5. $m\overset{\frown}{BA}$ 6. $m\overset{\frown}{BC}$

7. $m\overset{\frown}{CD}$ 8. $m\overset{\frown}{ACB}$

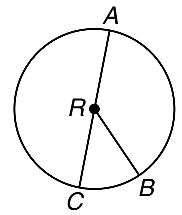
9. $m\overset{\frown}{BCD}$ 10. $m\overset{\frown}{AD}$

10-2 Study Guide and Intervention *(continued)*

Angles and Arcs

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

Example **In ⊙R, $m\angle ARB = 135$, $RB = 8$, and $\overline{AC}$ is a diameter. Find the length of $\widehat{AB}$.**



$m\angle ARB = 135$, so $m\widehat{AB} = 135$. Using the formula $C = 2\pi r$, the circumference is $2\pi(8)$ or 16π. To find the length of $\widehat{AB}$, write a proportion to compare each part to its whole.

$\dfrac{\text{length of } \widehat{AB}}{\text{circumference}} = \dfrac{\text{degree measure of arc}}{\text{degree measure of circle}}$ Proportion

$\dfrac{\ell}{16\pi} = \dfrac{135}{360}$ Substitution

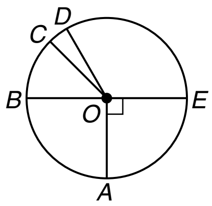
$\ell = \dfrac{(16\pi)(135)}{360}$ Multiply each side by 16π.

$= 6\pi$ Simplify.

The length of $\widehat{AB}$ is 6π or about 18.85 units.

Exercises

The diameter of ⊙O is 24 units long. Find the length of each arc for the given angle measure.



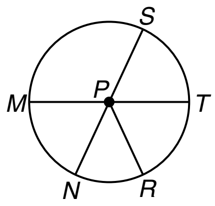
1. $\widehat{DE}$ if $m\angle DOE = 120$

2. $\widehat{DEA}$ if $m\angle DOE = 120$

3. $\widehat{BC}$ if $m\angle COB = 45$

4. $\widehat{CBA}$ if $m\angle COB = 45$

The diameter of ⊙P is 15 units long and $\angle SPT \cong \angle RPT$. Find the length of each arc for the given angle measure.



5. $\widehat{RT}$ if $m\angle SPT = 70$

6. $\widehat{NR}$ if $m\angle RPT = 50$

7. $\widehat{MST}$

8. $\widehat{MRS}$ if $m\angle MPS = 140$

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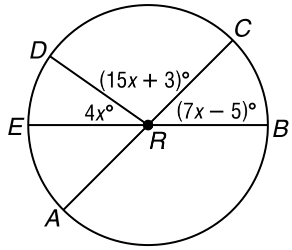
10-2 Skills Practice

Angles and Arcs

ALGEBRA In ⊙*R*, $\overline{AC}$ and $\overline{EB}$ are diameters. Find each measure.



1. $m\angle ERD$

2. $m\angle CRD$

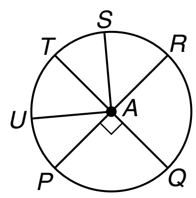
3. $m\angle BRC$

4. $m\angle ARB$

5. $m\angle ARE$

6. $m\angle BRD$

In ⊙*A*, $m\angle PAU = 40$, $\angle PAU \cong \angle SAT$, and $\angle RAS \cong \angle TAU$. Find each measure.



7. $m\overparen{PQ}$

8. $m\overparen{PQR}$

9. $m\overparen{ST}$

10. $m\overparen{RS}$

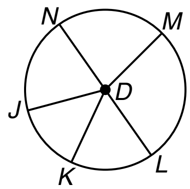
11. $m\overparen{RSU}$

12. $m\overparen{STP}$

13. $m\overparen{PQS}$

14. $m\overparen{PRU}$

The diameter of ⊙*D* is 18 units long. Find the length of each arc for the given angle measure.



15. $\overparen{LM}$ if $m\angle LDM = 100$

16. $\overparen{MN}$ if $m\angle MDN = 80$

17. $\overparen{KL}$ if $m\angle KDL = 60$

18. $\overparen{NJK}$ if $m\angle NDK = 120$

19. $\overparen{KLM}$ if $m\angle KDM = 160$

20. $\overparen{JK}$ if $m\angle JDK = 50$

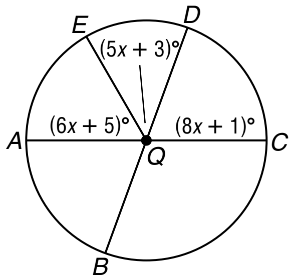
Lesson 10-2

NAME ______________________________ DATE ____________ PERIOD _____

10-2 Practice

Angles and Arcs

ALGEBRA In $\odot Q$, $\overline{AC}$ and $\overline{BD}$ are diameters. Find each measure.



1. $m\angle AQE$

2. $m\angle DQE$

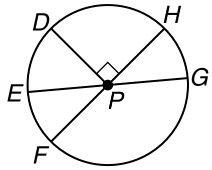
3. $m\angle CQD$

4. $m\angle BQC$

5. $m\angle CQE$

6. $m\angle AQD$

In $\odot P$, $m\angle GPH = 38$. Find each measure.



7. $m\overparen{EF}$

8. $m\overparen{DE}$

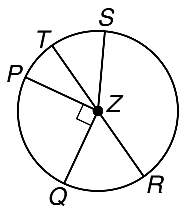
9. $m\overparen{FG}$

10. $m\overparen{DHG}$

11. $m\overparen{DFG}$

12. $m\overparen{DGE}$

The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.



13. $\overparen{QPT}$ if $m\angle QZT = 120$

14. $\overparen{QR}$ if $m\angle QZR = 60$

15. $\overparen{PQR}$ if $m\angle PZR = 150$

16. $\overparen{QPS}$ if $m\angle QZS = 160$

HOMEWORK For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

Homework	
Less than 1 hour	8%
1–2 hours	29%
2–3 hours	58%
3–4 hours	3%
Over 4 hours	2%

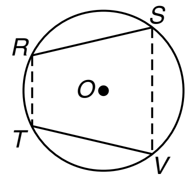
17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?

18. Describe the arcs associated with each category.

10-3 Study Guide and Intervention

Arcs and Chords

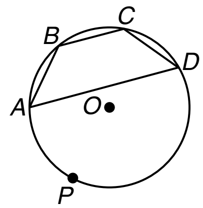
Arcs and Chords Points on a circle determine both chords and arcs. Several properties are related to points on a circle.



$\widehat{RS} \cong \widehat{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.
RSVT is inscribed in ⊙*O*.
⊙*O* is circumscribed about *RSVT*.

- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.

Example **Trapezoid *ABCD* is inscribed in ⊙*O*. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $m\widehat{BC} = 50$, what is $m\widehat{APD}$?**



Chords $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are congruent, so $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CD}$ are congruent. $m\widehat{BC} = 50$, so $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} = 50 + 50 + 50 = 150$. Then $m\widehat{APD} = 360 - 150$ or 210.

Exercises

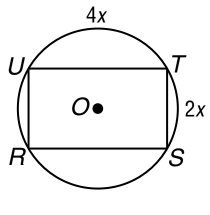
Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.

1. hexagon **2.** pentagon **3.** triangle

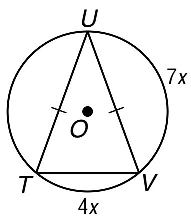
4. square **5.** octagon **6.** 36-gon

Determine the measure of each arc of the circle circumscribed about the polygon.

7.



8.



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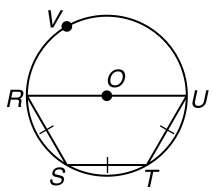


Lesson 10-3

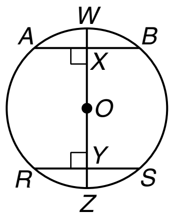
NAME ____ DATE ____________ PERIOD _____

10-3 Study Guide and Intervention *(continued)*

Arcs and Chords

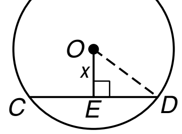
Diameters and Chords

- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\overset{\frown}{AW} \cong \overset{\frown}{WB}$.
If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.
If $\overline{AB} \cong \overline{RS}$, then $\overline{AB}$ and $\overline{RS}$ are equidistant from point O.

Example **In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x.**



A diameter or radius perpendicular to a chord bisects the chord, so ED is half of CD.

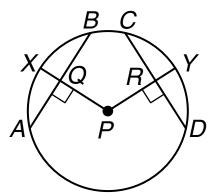
$$ED = \frac{1}{2}(24)$$
$$= 12$$

Use the Pythagorean Theorem to find x in $\triangle OED$.

$(OE)^2 + (ED)^2 = (OD)^2$	Pythagorean Theorem
$x^2 + 12^2 = 15^2$	Substitution
$x^2 + 144 = 225$	Multiply.
$x^2 = 81$	Subtract 144 from each side.
$x = 9$	Take the square root of each side.

Exercises

In $\odot P$, $CD = 24$ and $m\overset{\frown}{CY} = 45$. Find each measure.

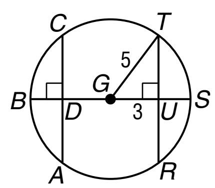


1. AQ **2.** RC **3.** QB

4. AB **5.** $m\overset{\frown}{DY}$ **6.** $m\overset{\frown}{AB}$

7. $m\overset{\frown}{AX}$ **8.** $m\overset{\frown}{XB}$ **9.** $m\overset{\frown}{CD}$

In $\odot G$, $DG = GU$ and $AC = RT$. Find each measure.



10. TU **11.** TR **12.** $m\overset{\frown}{TS}$

13. CD **14.** GD **15.** $m\overset{\frown}{AB}$

16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius.

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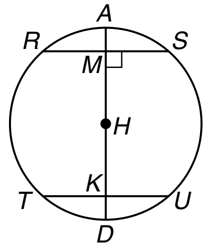
10-3 Skills Practice

Arcs and Chords

In $\odot H$, $m\widehat{RS} = 82$, $m\widehat{TU} = 82$, $RS = 46$, and $\overline{TU} \cong \overline{RS}$. Find each measure.



1. TU

2. TK

3. MS

4. $m\angle HKU$

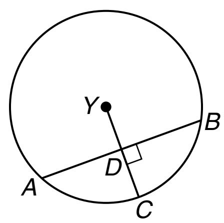
5. $m\widehat{AS}$

6. $m\widehat{AR}$

7. $m\widehat{TD}$

8. $m\widehat{DU}$

The radius of $\odot Y$ is 34, $AB = 60$, and $m\widehat{AC} = 71$. Find each measure.



9. $m\widehat{BC}$

10. $m\widehat{AB}$

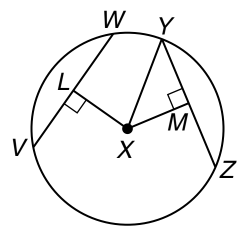
11. AD

12. BD

13. YD

14. DC

In $\odot X$, $LX = MX$, $XY = 58$, and $VW = 84$. Find each measure.



15. YZ

16. YM

17. MX

18. MZ

19. LV

20. LX

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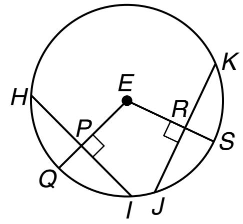
10-3 Practice

Arcs and Chords

In ⊙E, $m\overset{\frown}{HQ} = 48$, $HI = JK$, and $JR = 7.5$. Find each measure.



1. $m\overset{\frown}{HI}$

2. $m\overset{\frown}{QI}$

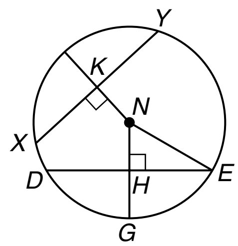
3. $m\overset{\frown}{JK}$

4. HI

5. PI

6. JK

The radius of ⊙N is 18, $NK = 9$, and $m\overset{\frown}{DE} = 120$. Find each measure.



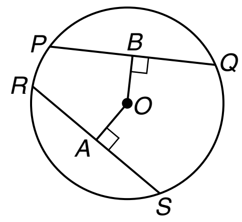
7. $m\overset{\frown}{GE}$

8. $m\angle HNE$

9. $m\angle HEN$

10. HN

The radius of ⊙O = 32, $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$, and $PQ = 56$. Find each measure.



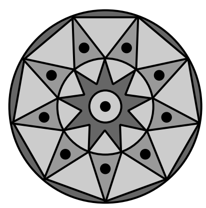
11. PB

14. BQ

12. OB

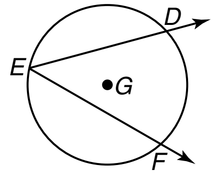
16. RS

13. MANDALAS The base figure in a mandala design is a nine-pointed star. Find the measure of each arc of the circle circumscribed about the star.



10-4 Study Guide and Intervention

Inscribed Angles



$m\angle DEF = \frac{1}{2}m\widehat{DF}$

Inscribed Angles An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In $\odot G$, inscribed $\angle DEF$ **intercepts** $\widehat{DF}$.

Inscribed Angle Theorem	If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.

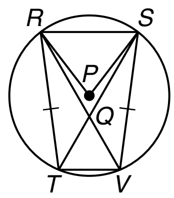
Example **In $\odot G$ above, $m\widehat{DF} = 90$. Find $m\angle DEF$.**

$\angle DEF$ is an inscribed angle so its measure is half of the intercepted arc.

$$m\angle DEF = \frac{1}{2}m\widehat{DF}$$
$$= \frac{1}{2}(90) \text{ or } 45$$

Exercises

Use $\odot P$ for Exercises 1–10. In $\odot P$, $\overline{RS} \parallel \overline{TV}$ and $\overline{RT} \cong \overline{SV}$.



1. Name the intercepted arc for $\angle RTS$.

2. Name an inscribed angle that intercepts $\widehat{SV}$.

In $\odot P$, $m\widehat{SV} = 120$ and $m\angle RPS = 76$. Find each measure.

3. $m\angle PRS$

4. $m\widehat{RSV}$

5. $m\widehat{RT}$

6. $m\angle RVT$

7. $m\angle QRS$

8. $m\angle STV$

9. $m\widehat{TV}$

10. $m\angle SVT$

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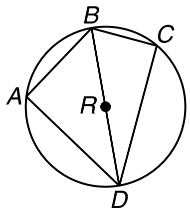
10-4 Study Guide and Intervention *(continued)*

Inscribed Angles

Angles of Inscribed Polygons

An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.



- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle.

  If $\widehat{BCD}$ is a semicircle, then $m\angle BCD = 90$.

- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

  For inscribed quadrilateral $ABCD$,
  $m\angle A + m\angle C = 180$ and
  $m\angle ABC + m\angle ADC = 180$.

Example **In $\odot R$ above, $BC = 3$ and $BD = 5$. Find each measure.**

a. $m\angle C$

$\angle C$ intercepts a semicircle. Therefore $\angle C$ is a right angle and $m\angle C = 90$.

b. CD

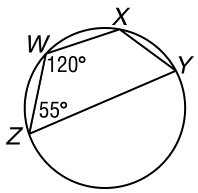
$\triangle BCD$ is a right triangle, so use the Pythagorean Theorem to find CD.

$$(CD)^2 + (BC)^2 = (BD)^2$$
$$(CD)^2 + 3^2 = 5^2$$
$$(CD)^2 = 25 - 9$$
$$(CD)^2 = 16$$
$$CD = 4$$

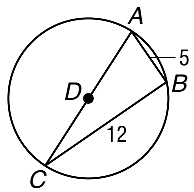
Exercises

Find the measure of each angle or segment for each figure.

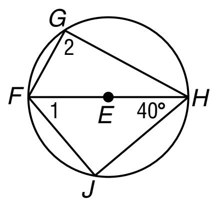
1. $m\angle X$, $m\angle Y$



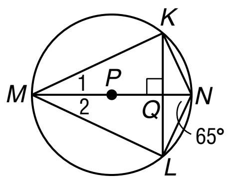
2. AD



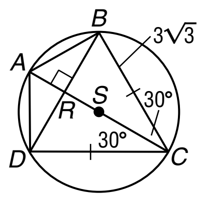
3. $m\angle 1$, $m\angle 2$



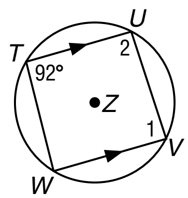
4. $m\angle 1$, $m\angle 2$



5. AB, AC



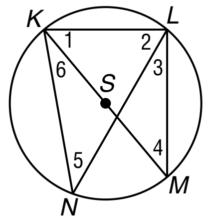
6. $m\angle 1$, $m\angle 2$



10-4 Skills Practice

Inscribed Angles

In ⊙S, $m\widehat{KL} = 80$, $m\widehat{LM} = 100$, and $m\widehat{MN} = 60$. Find the measure of each angle.



1. $m\angle 1$

2. $m\angle 2$

3. $m\angle 3$

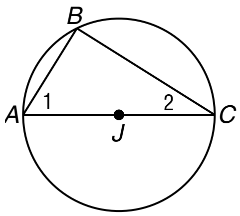
4. $m\angle 4$

5. $m\angle 5$

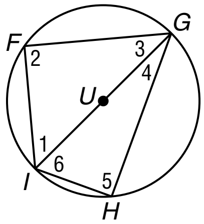
6. $m\angle 6$

ALGEBRA Find the measure of each numbered angle.

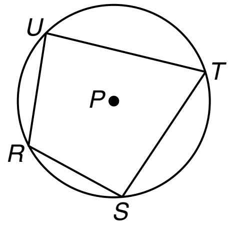
7. $m\angle 1 = 5x - 2$, $m\angle 2 = 2x + 8$



8. $m\angle 1 = 5x$, $m\angle 3 = 3x + 10$, $m\angle 4 = y + 7$, $m\angle 6 = 3y + 11$



Quadrilateral $RSTU$ is inscribed in ⊙P such that $m\widehat{STU} = 220$ and $m\angle S = 95$. Find each measure.



9. $m\angle R$

10. $m\angle T$

11. $m\angle U$

12. $m\widehat{SRU}$

13. $m\widehat{RUT}$

14. $m\widehat{RST}$

Lesson 10-4

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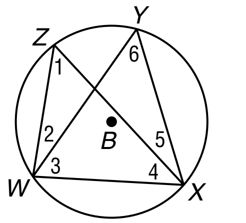
10-4 Practice

Inscribed Angles

In $\odot B$, $m\widehat{WX} = 104$, $m\widehat{WZ} = 88$, and $m\angle ZWY = 26$. Find the measure of each angle.



1. $m\angle 1$

2. $m\angle 2$

3. $m\angle 3$

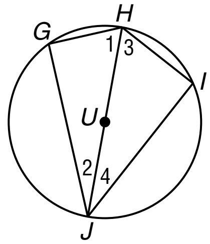
4. $m\angle 4$

5. $m\angle 5$

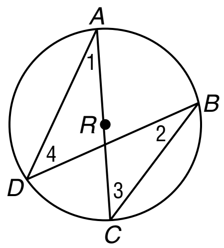
6. $m\angle 6$

ALGEBRA Find the measure of each numbered angle.

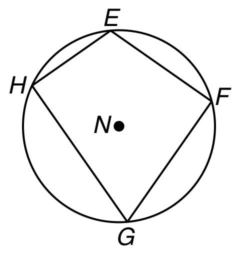
7. $m\angle 1 = 5x + 2$, $m\angle 2 = 2x - 3$,
$m\angle 3 = 7y - 1$, $m\angle 4 = 2y + 10$



8. $m\angle 1 = 4x - 7$, $m\angle 2 = 2x + 11$,
$m\angle 3 = 5y - 14$, $m\angle 4 = 3y + 8$



Quadrilateral $EFGH$ is inscribed in $\odot N$ such that $m\widehat{FG} = 97$, $m\widehat{GH} = 117$, and $m\widehat{EHG} = 164$. Find each measure.



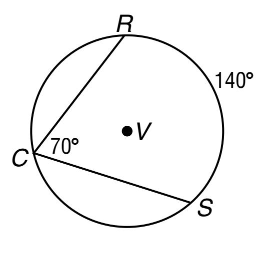
9. $m\angle E$

10. $m\angle F$

11. $m\angle G$

12. $m\angle H$

13. PROBABILITY In $\odot V$, point C is randomly located so that it does not coincide with points R or S. If $m\widehat{RS} = 140$, what is the probability that $m\angle RCS = 70$?



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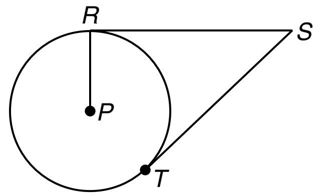
10-5 Study Guide and Intervention

Tangents

Tangents A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

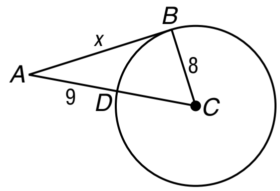
- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.



$\overline{RP} \perp \overline{SR}$ if and only if $\overline{SR}$ is tangent to $\odot P$.

If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

Example **$\overline{AB}$ is tangent to $\odot C$. Find x.**



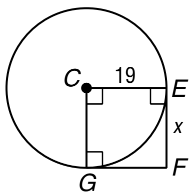
$\overline{AB}$ is tangent to $\odot C$, so $\overline{AB}$ is perpendicular to radius $\overline{BC}$. $\overline{CD}$ is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17. Use the Pythagorean Theorem with right $\triangle ABC$.

$(AB)^2 + (BC)^2 = (AC)^2$	Pythagorean Theorem
$x^2 + 8^2 = 17^2$	Substitution
$x^2 + 64 = 289$	Multiply.
$x^2 = 225$	Subtract 64 from each side.
$x = 15$	Take the square root of each side.

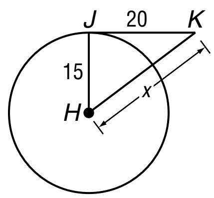
Exercises

Find x. Assume that segments that appear to be tangent are tangent.

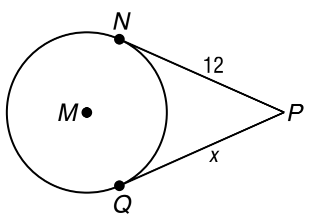
1.



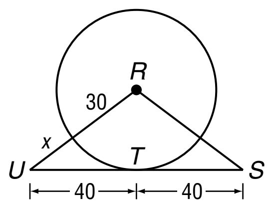
2.



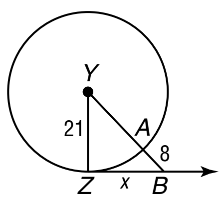
3.



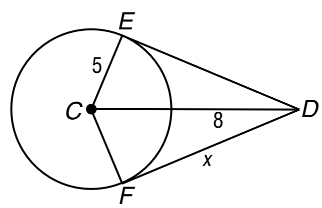
4.



5.



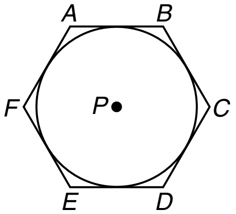
6.



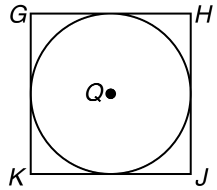
10-5 Study Guide and Intervention *(continued)*

Tangents

Circumscribed Polygons When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.

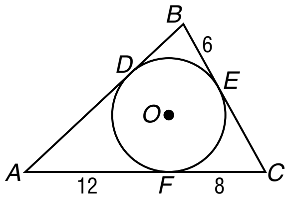


Hexagon *ABCDEF* is circumscribed about $\odot P$.
$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$ are tangent to $\odot P$.



Square *GHJK* is circumscribed about $\odot Q$.
$\overline{GH}$, $\overline{JH}$, $\overline{JK}$, and $\overline{KG}$ are tangent to $\odot Q$.

Example **$\triangle ABC$ is circumscribed about $\odot O$. Find the perimeter of $\triangle ABC$.**



$\triangle ABC$ is circumscribed about $\odot O$, so points D, E, and F are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.

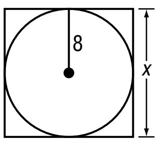
$$\begin{aligned} P &= AD + AF + BE + BD + CF + CE \\ &= 12 + 12 + 6 + 6 + 8 + 8 \\ &= 52 \end{aligned}$$

The perimeter is 52 units.

Exercises

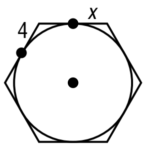
Find *x*. Assume that segments that appear to be tangent are tangent.

1.



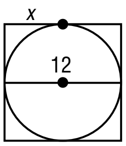
square

2.



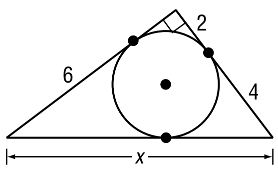
regular hexagon

3.

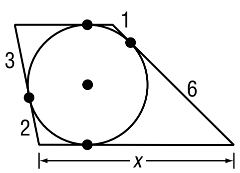


square

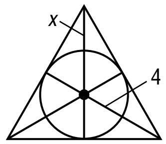
4.



5.



6.



equilateral triangle

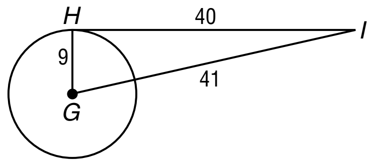
NAME ______________________________ DATE ____________ PERIOD _____

10-5 Skills Practice

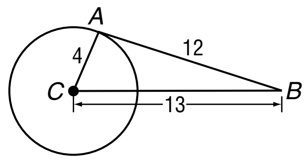
Tangents

Determine whether each segment is tangent to the given circle.

1. $\overline{HI}$

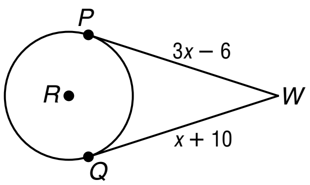


2. $\overline{AB}$

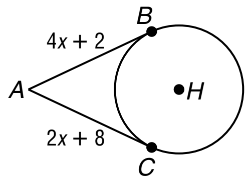


Find *x*. Assume that segments that appear to be tangent are tangent.

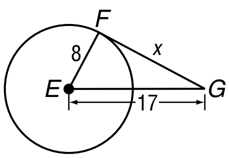
3.



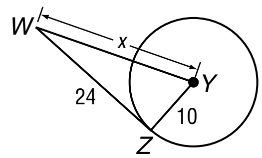
4.



5.

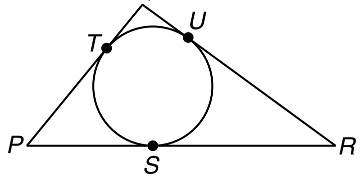


6.

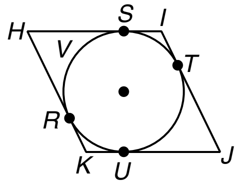


Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

7. $QT = 4$, $PT = 9$, $SR = 13$



8. $HIJK$ is a rhombus, $SI = 5$, $HR = 13$



Lesson 10-5

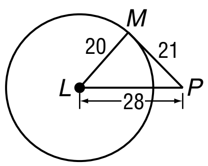
NAME _______________________ DATE ____________ PERIOD _____

10-5 Practice

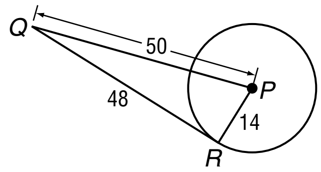
Tangents

Determine whether each segment is tangent to the given circle.

1. $\overline{MP}$

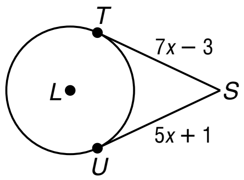


2. $\overline{QR}$

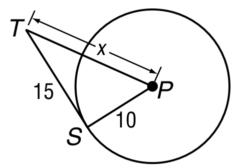


Find *x*. Assume that segments that appear to be tangent are tangent.

3.

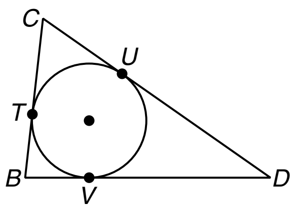


4.

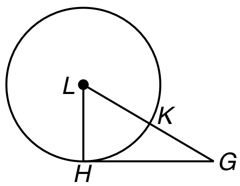


Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

5. $CD = 52$, $CU = 18$, $TB = 12$

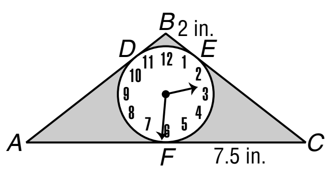


6. $KG = 32$, $HG = 56$



CLOCKS For Exercises 7 and 8, use the following information.

The design shown in the figure is that of a circular clock face inscribed in a triangular base. *AF* and *FC* are equal.



7. Find *AB*.

8. Find the perimeter of the clock.

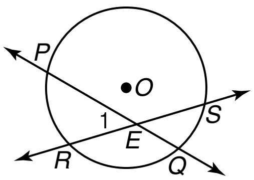
NAME ____ DATE ____________ PERIOD _____

10-6 Study Guide and Intervention

Secants, Tangents, and Angle Measures

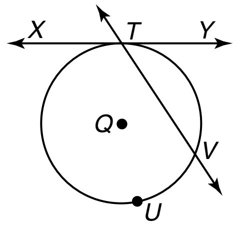
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

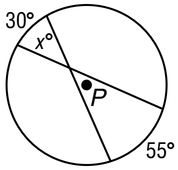


$m\angle XTV = \frac{1}{2}m\widehat{TUV}$

$m\angle YTV = \frac{1}{2}m\widehat{TV}$

Example 1 **Find *x*.**

The two secants intersect inside the circle, so x is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.



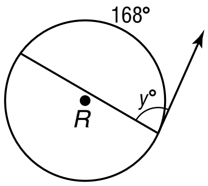
$x = \frac{1}{2}(30 + 55)$

$= \frac{1}{2}(85)$

$= 42.5$

Example 2 **Find *y*.**

The secant and the tangent intersect at the point of tangency, so the measure the angle is one-half the measure of its intercepted arc.



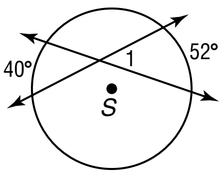
$y = \frac{1}{2}(168)$

$= 84$

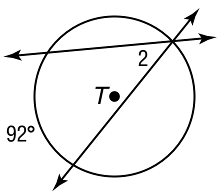
Exercises

Find each measure.

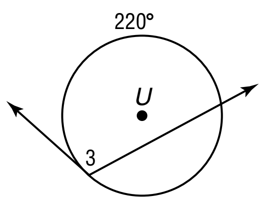
1. $m\angle 1$



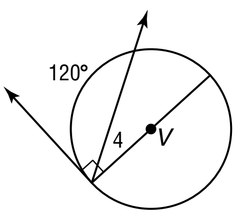
2. $m\angle 2$



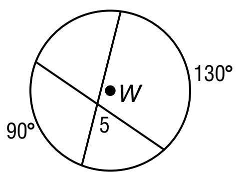
3. $m\angle 3$



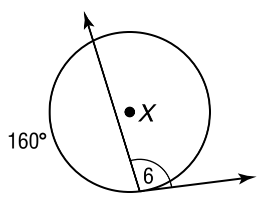
4. $m\angle 4$



5. $m\angle 5$



6. $m\angle 6$

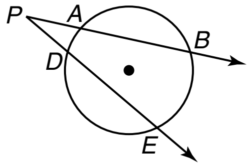


10-6 Study Guide and Intervention *(continued)*

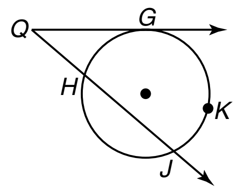
Secants, Tangents, and Angle Measures

Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

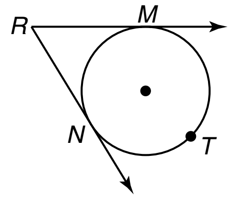
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.



$\overrightarrow{PB}$ and $\overrightarrow{PE}$ are secants.
$m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$

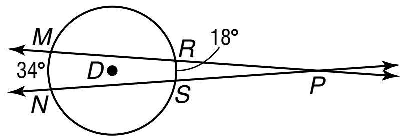


$\overrightarrow{QG}$ is a tangent. $\overrightarrow{QJ}$ is a secant.
$m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



$\overrightarrow{RM}$ and $\overrightarrow{RN}$ are tangents.
$m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MN})$

Example **Find $m\angle MPN$.**



$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.

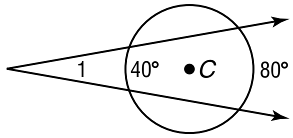
$$m\angle MPN = \frac{1}{2}(m\widehat{MN} - m\widehat{RS})$$
$$= \frac{1}{2}(34 - 18)$$
$$= \frac{1}{2}(16) \text{ or } 8$$

The measure of the angle is 8.

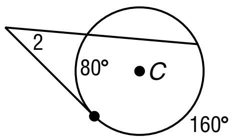
Exercises

Find each measure.

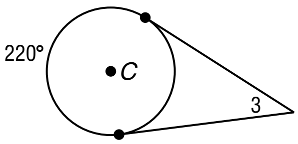
1. $m\angle 1$



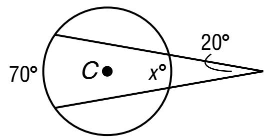
2. $m\angle 2$



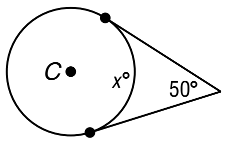
3. $m\angle 3$



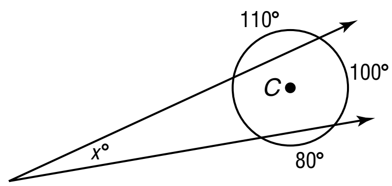
4. x



5. x



6. x



NAME ______________________________ DATE ____________ PERIOD _____

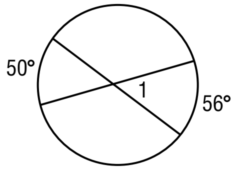
10-6 Skills Practice

Secants, Tangents, and Angle Measures

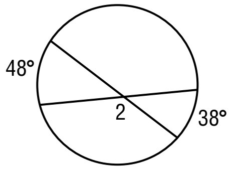
Lesson 10-6

Find each measure.

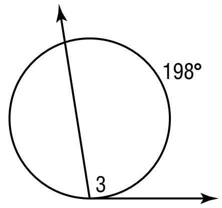
1. $m\angle 1$



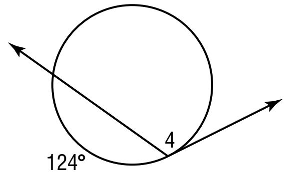
2. $m\angle 2$



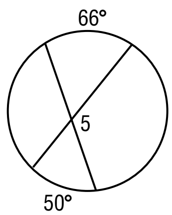
3. $m\angle 3$



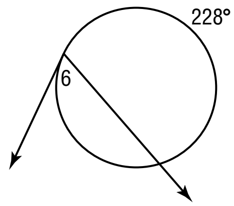
4. $m\angle 4$



5. $m\angle 5$

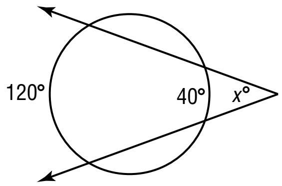


6. $m\angle 6$

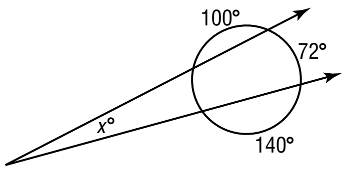


Find *x*. Assume that any segment that appears to be tangent is tangent.

7.



8.



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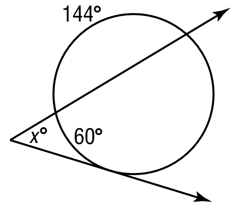
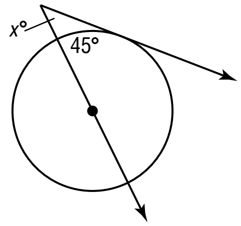


10.



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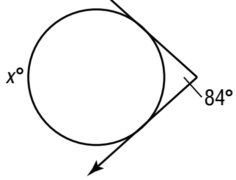

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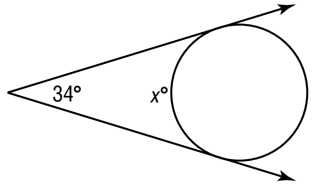


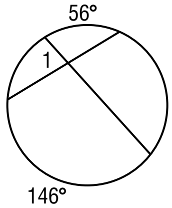
NAME ______________________________ DATE ____________ PERIOD _____

10-6 Practice

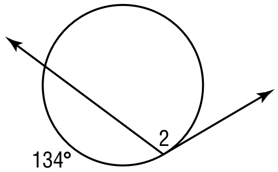
Secants, Tangents, and Angle Measures

Find each measure.

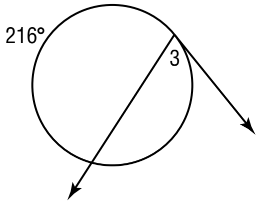
1. $m\angle 1$



2. $m\angle 2$

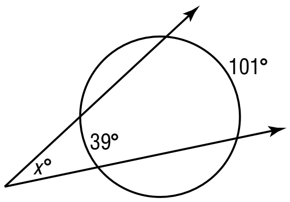


3. $m\angle 3$

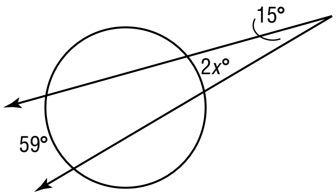


Find *x*. Assume that any segment that appears to be tangent is tangent.

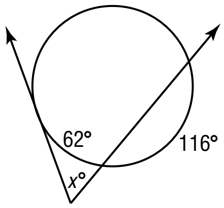
7.



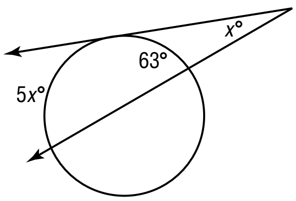
8.



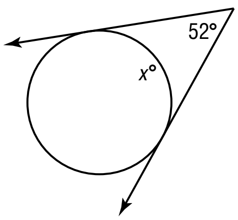
9.



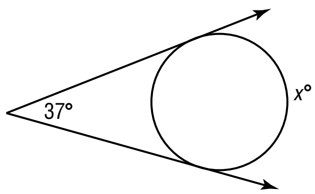
10.



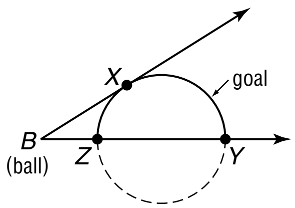
11.



12.



9. RECREATION In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.

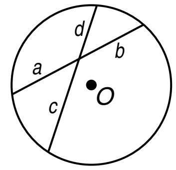


NAME ____ DATE ____________ PERIOD _____

10-7 Study Guide and Intervention

Special Segments in a Circle

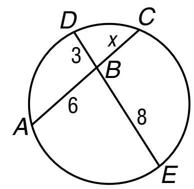
Segments Intersecting Inside a Circle If two chords intersect in a circle, then the products of the measures of the chords are equal.



$a \cdot b = c \cdot d$

Example **Find *x*.**

The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.



$AB \cdot BC = EB \cdot BD$

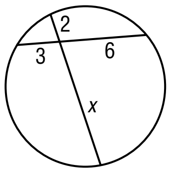
$6 \cdot x = 8 \cdot 3$	Substitution
$6x = 24$	Simplify.
$x = 4$	Divide each side by 6.

$AB \cdot BC = EB \cdot BD$

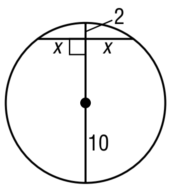
Exercises

Find *x* to the nearest tenth.

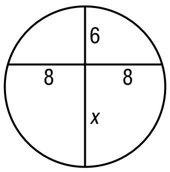
1.



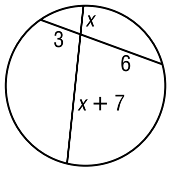
2.



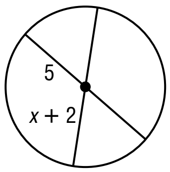
3.



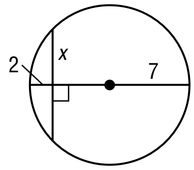
4.



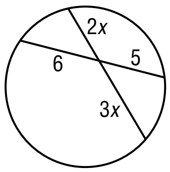
5.



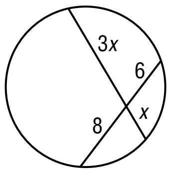
6.



7.



8.

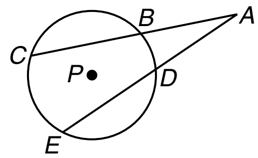


10-7 Study Guide and Intervention *(continued)*

Special Segments in a Circle

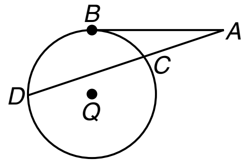
Segments Intersecting Outside a Circle If secants and tangents intersect outside a circle, then two products are equal.

- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



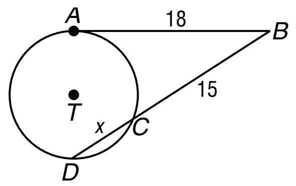
$\overline{AC}$ and $\overline{AE}$ are secant segments.
$\overline{AB}$ and $\overline{AD}$ are external secant segments.
$AC \cdot AB = AE \cdot AD$

- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$\overline{AB}$ is a tangent segment.
$\overline{AD}$ is a secant segment.
$\overline{AC}$ is an external secant segment.
$(AB)^2 = AD \cdot AC$

Example **Find x to the nearest tenth.**

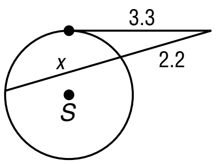


The tangent segment is $\overline{AB}$, the secant segment is $\overline{BD}$, and the external secant segment is $\overline{BC}$.

$(AB)^2 = BC \cdot BD$
$(18)^2 = 15(15 + x)$
$324 = 225 + 15x$
$99 = 15x$
$6.6 = x$

Exercises

Find x to the nearest tenth. Assume segments that appear to be tangent are tangent.

1.


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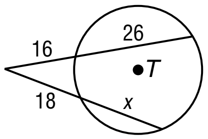

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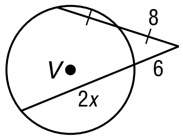

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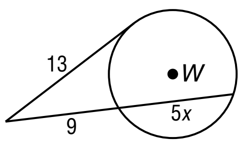

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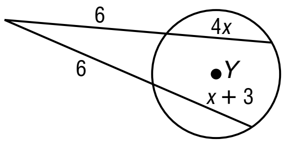

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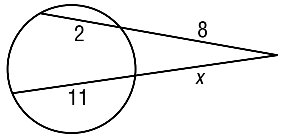

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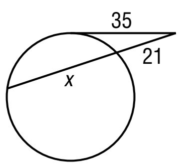

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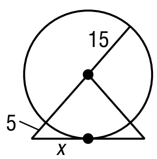

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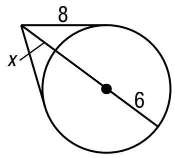
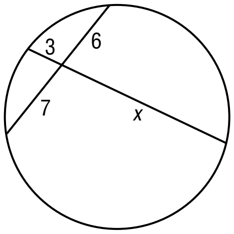


10-7 Skills Practice

Special Segments in a Circle

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

1.



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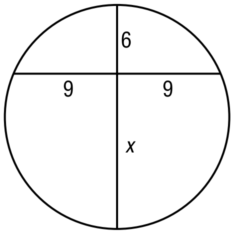

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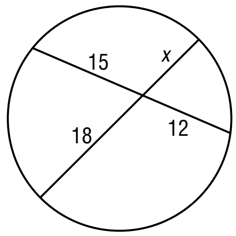

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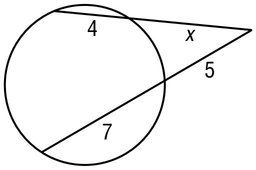

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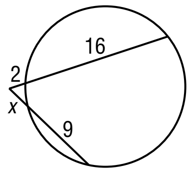

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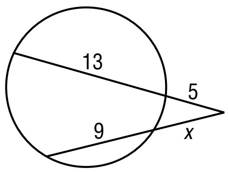

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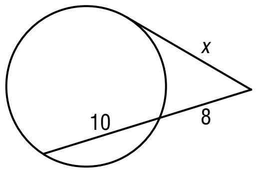

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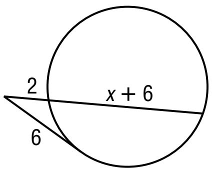

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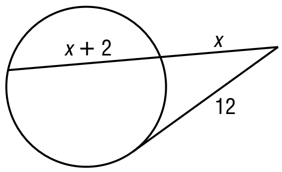

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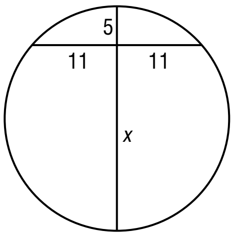
10-7 Practice

Special Segments in a Circle

Find *x* to the nearest tenth. Assume that segments that appear to be tangent are tangent.

1.



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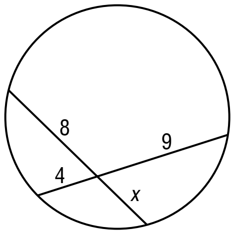

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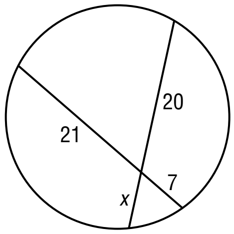

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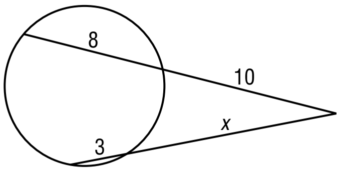

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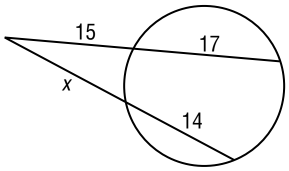

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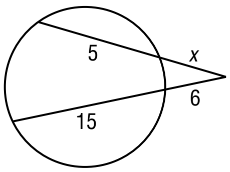

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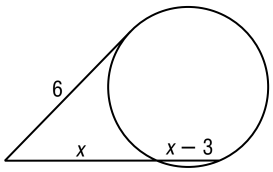

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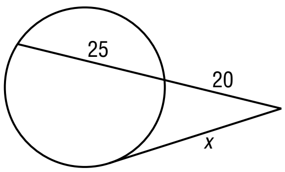

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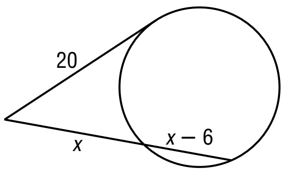
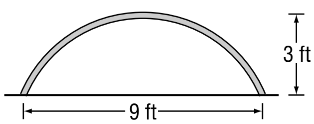


10. CONSTRUCTION An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch.



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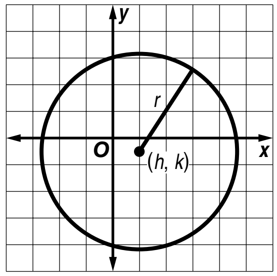
10-8 Study Guide and Intervention

Equations of Circles

Equation of a Circle A **circle** is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.



Standard Equation of a Circle	An equation for a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle with center (−1, 3) and radius 6.**

Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - (-1))^2 + (y - 3)^2 = 6^2$ Substitution

$(x + 1)^2 + (y - 3)^2 = 36$ Simplify.

Exercises

Write an equation for each circle.

1. center at (0, 0), $r = 8$

2. center at $(-2, 3)$, $r = 5$

3. center at $(2, -4)$, $r = 1$

4. center at $(-1, -4)$, $r = 2$

5. center at $(-2, -6)$, diameter = 8

6. center at $\left(-\frac{1}{2}, \frac{1}{4}\right)$, $r = \sqrt{3}$

7. center at the origin, diameter = 4

8. center at $\left(1, -\frac{5}{8}\right)$, $r = \sqrt{5}$

9. Find the center and radius of a circle with equation $x^2 + y^2 = 20$.

10. Find the center and radius of a circle with equation $(x + 4)^2 + (y + 3)^2 = 16$.

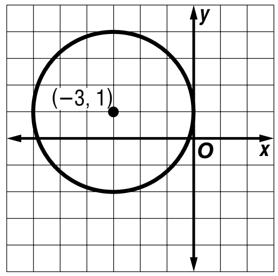
Lesson 10-8

10-8 Study Guide and Intervention *(continued)*

Equations of Circles

Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example **Graph $(x + 3)^2 + (y - 1)^2 = 9$.**



Use the parts of the equation to find (h, k) and r.

$(x - h)^2 + (y - k)^2 = r^2$

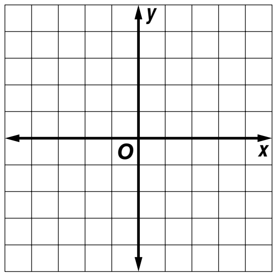
$$\begin{array}{lll}
(x - h)^2 = (x + 3)^2 & (y - k)^2 = (y - 1)^2 & r^2 = 9 \\
x - h = x + 3 & y - k = y - 1 & r = 3 \\
-h = 3 & -k = -1 & \\
h = -3 & k = 1 &
\end{array}$$

The center is at $(-3, 1)$ and the radius is 3. Graph the center.
Use a compass set at a radius of 3 grid squares to draw the circle.

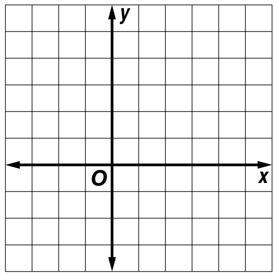
Exercises

Graph each equation.

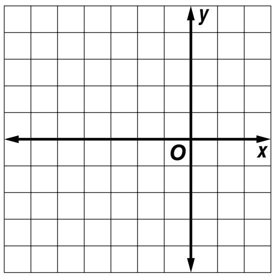
1. $x^2 + y^2 = 16$



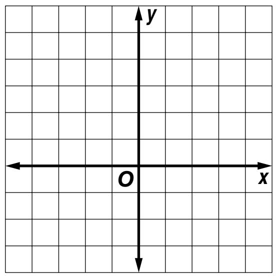
2. $(x - 2)^2 + (y - 1)^2 = 9$



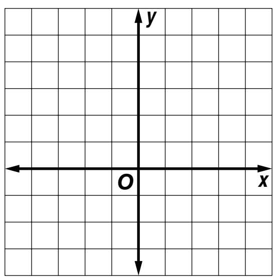
3. $(x + 2)^2 + y^2 = 16$



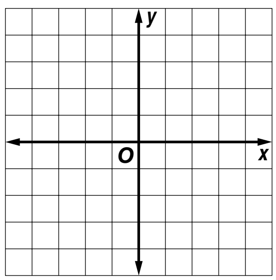
4. $(x + 1)^2 + (y - 2)^2 = 6.25$



5. $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = 4$



6. $x^2 + (y - 1)^2 = 9$



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10-8 Skills Practice

Equations of Circles

Write an equation for each circle.

1. center at origin, $r = 6$

2. center at $(0, 0)$, $r = 2$

3. center at $(4, 3)$, $r = 9$

4. center at $(7, 1)$, $d = 24$

5. center at $(-5, 2)$, $r = 4$

6. center at $(6, -8)$, $d = 10$

7. a circle with center at $(8, 4)$ and a radius with endpoint $(0, 4)$

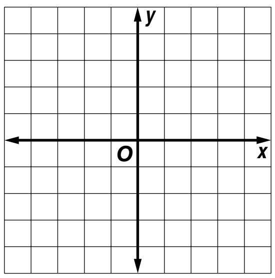
8. a circle with center at $(-2, -7)$ and a radius with endpoint $(0, 7)$

9. a circle with center at $(-3, 9)$ and a radius with endpoint $(1, 9)$

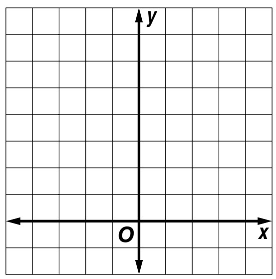
10. a circle whose diameter has endpoints $(-3, 0)$ and $(3, 0)$

Graph each equation.

11. $x^2 + y^2 = 16$



12. $(x - 1)^2 + (y - 4)^2 = 9$



NAME ____ DATE ____________ PERIOD _____

10-8 Practice

Equations of Circles

Write an equation for each circle.

1. center at origin, $r = 7$

2. center at $(0, 0)$, $d = 18$

3. center at $(-7, 11)$, $r = 8$

4. center at $(12, -9)$, $d = 22$

5. center at $(-6, -4)$, $r = \sqrt{5}$

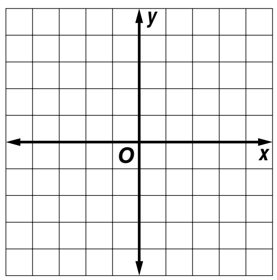
6. center at $(3, 0)$, $d = 28$

7. a circle with center at $(-5, 3)$ and a radius with endpoint $(2, 3)$

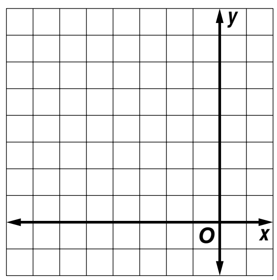
8. a circle whose diameter has endpoints $(4, 6)$ and $(-2, 6)$

Graph each equation.

9. $x^2 + y^2 = 4$



10. $(x + 3)^2 + (y - 3)^2 = 9$



11. EARTHQUAKES When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake.

